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On social inequality: Analyzing the rich–poor disparity

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HIGHLIGHTS

- A Lorenz-based quantitative analysis of the rich–poor disparity is presented.
- A global measure of socioeconomic inequality, the "median index", is devised.
- A local measure of socioeconomic inequality, the "power-law index", is devised.
- These indices facilitate the empirical tracking of the rich–poor disparity.

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ABSTRACT

From the Old Testament to the Communist Manifesto, and from the French Revolution to the Occupy Wall Street protests, social inequality has always been at the focal point of public debate, as well as a major driver of political change. Although being of prime interest since Biblical times, the scientific investigation of the distributions of wealth and income in human societies began only at the close of the nineteenth century, and was pioneered by Pareto, Lorenz, Gini, and Pietra. The methodologies introduced by these trailblazing scholars form the bedrock of the contemporary science of social inequality. Based on this bedrock we present a new quantitative approach to the analysis of wealth and income distributions, which sets its spotlight on the most heated facet of the current global debate on social inequality—the rich–poor disparity. Our approach offers researchers highly applicable quantitative tools to empirically track and statistically analyze the growing gap between the rich and the poor.

1. Introduction

Since the seminal work of the Italian economist Vilfredo Pareto [1], the distributions of wealth and income in human societies have been extensively studied [2–4], as well as the social inequality reflected by these distributions [5–7]. Recently, protest movements such as *Occupy Wall Street* raised the issue of social inequality to the spotlight of the global public debate [8–10]. Consequently, quantitative methodologies for the analysis of social inequality – and especially measures capturing the growing disparity between the rich and the poor – are timely and of significance.

In this paper we establish a new quantitative approach to the analysis of social inequality, which sets its main focus on the rich–poor disparity. The paper begins, in Section 2, with a short review of the common methodology of the contemporary science of social inequality—which is due to the works of Pareto [1], Lorenz [11], Gini [12], and Pietra [13]. Sections 3 and 4 present our quantitative methodology—Section 3 from an additive perspective, and Section 4 from a multiplicative perspective. Section 5 combines our methodology with the empirical fact – originally due to Pareto [1] – that the probability

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density functions quantifying the distributions of wealth and income in human societies are governed by power-law tails [2–4]. Section 6 illustrates the application of the methodology with two specific examples – the Inverse Pareto and Pareto probability laws – which govern, asymptotically, the tails of the distributions of wealth and income in human societies. The approach presented in this paper offers researchers highly applicable quantitative tools to empirically track and statistically analyze the growing gap between the rich and the poor.

2. Lorenz curves and inequality indices

The common quantitative methodology regarding the distribution of wealth (and income) in human societies was devised by the American statistician Max Lorenz [11]. The *Lorenz curve* $y = L_<(x)$ $(0 \le x, y \le 1)$ of a given human society reads out as follows: the *low* 100x% of the society's population are in possession of 100y% of the society's overall wealth. Also, the *Lorenz curve* $y = L_>(x)$ $(0 \le x, y \le 1)$ of the same society reads out as: the *top* 100x% of the society's population are in possession of 100y% of the society's overall wealth. It is straightforward to note the following connection between the two Lorenz curves:

$$L_< (1 - x) + L_> (x) = 1 \tag{1}$$

$(0 \le x \le 1)$. The graphs of both Lorenz curves $L_<(x)$ and $L_>(x)$ reside in the unit square, and increase monotonically from the origin $(0, 0)$ to the point $(1, 1)$; the Lorenz curve $L_<(x)$ is convex and the Lorenz curve $L_>(x)$ is concave.

Two diametric social extremes are pure communism and absolute monarchy. In pure communism the distribution of wealth is totally symmetric, i.e., the society's overall wealth is equally distributed among the society's population. Pure communism is characterized by the Lorenz curves $L_<(x) = L_>(x) = x$ $(0 \le x \le 1)$. The line $y = x$ $(0 \le x \le 1)$ – which characterizes pure communism in the space of Lorenz curves – is termed *the line of perfect equality*.

In absolute monarchy the distribution of wealth is totally asymmetric, i.e., the society's overall wealth is held exclusively by one single individual (the absolute monarch), leaving the rest of the society's population (everyone but the absolute monarch) completely impoverished. Absolute monarchy manifests a ''the winner takes it all'' scenario, and is characterized by the following degenerate and discontinuous Lorenz curves: (i) $L_<(x) = 0$ in the range $0 \le x < 1$, and $L_<(1) = 1$; and (ii) $L_>(0) = 0$ and $L_>(x) = 1$ in the range $0 < x \le 1$.

An *inequality index* I is a numerical measure of social inequality that takes values in the unit interval, assigns the value $I = 0$ in the case of pure communism, and assigns the value $I = 1$ in the case of absolute monarchy. The distances of the Lorenz curves $L_<(x)$ and $L_>(x)$ from the line of perfect equality are quantitative measures of the society's divergence from pure communism. Thus, after proper normalization, these distances can serve as inequality indices.

The most popular distance-based inequality index was introduced by the Italian statistician and demographer Corrado Gini [12]. The *Gini index* corresponding to the Lorenz curves $L_<(x)$ and $L_>(x)$ is defined as twice the area captured between these curves and the line of perfect equality:

$$I_G = 2\int_0^1 [x - L_< (x)]\, \mathrm{d}x = 2\int_0^1 [L_> (x) - x]\, \mathrm{d}x. \tag{2}$$

The factor 2 in Eq. (2) is set so as to attain normalization (indeed, in the case of absolute monarchy the integrals appearing on the middle part and on the right-hand-side of Eq. (2) equal 1/2).

A different inequality index was introduced by the Italian statistician Gaetano Pietra [13]. The *Pietra index* corresponding to the Lorenz curves $L_<(x)$ and $L_>(x)$ is the maximal vertical distance between these curves and the line of perfect equality:

$$I_P = \max_{0 \le x \le 1} [x - L_< (x)] = \max_{0 \le x \le 1} [L_> (x) - x] \,. \tag{3}$$

The Pietra index has a compelling intuitive ''Robin Hood'' meaning: it is the proportion of the society's overall wealth that has to be redistributed – i.e., transferred from the rich to the poor – in order to attain pure communism [14].

For a detailed exposition of Lorenz curves and inequality indices the readers are referred to Refs. [5–7]. We conclude this section with the introduction of yet another inequality index—which will turn out to play a key role in the sequel. To that end we set the points $m_<$ and $m_>$ to be given implicitly by $L_<(m_<) = 1/2$ and $L_>(m_>) = 1/2$. Namely, the points $m_<$ and $m_>$ are the ''medians'' of the corresponding Lorenz curves: the *low* 100$m_<$% of the society's population, as well as the *top* 100$m_>$% of the society's population, are in possession of *half* the society's overall wealth. In other words, $m_<$ and $m_>$ are the *breakeven points* between the society's upper and lower classes. Clearly, the Lorenz medians are coupled by the connection $m_< + m_> = 1$. Since the Lorenz curves are monotone increasing, inverting them yields the Lorenz medians explicitly: $m_< = L_<^{-1}(1/2)$ and $m_> = L_>^{-1}(1/2)$.

The Lorenz medians $m_<$ and $m_>$ take values in the intervals $[1/2, 1]$ and $[0, 1/2]$, respectively. In the case of pure communism the Lorenz curves coincide with the line of perfect equality and hence $m_< = m_> = 1/2$. On the other hand, in the case of absolute monarchy the degeneracy of the Lorenz curves implies that $m_< = 1$ and $m_> = 0$. Consequently, the Lorenz medians yield the following inequality index—which we term the *median index*:

$$I_M = m_< - m_>. \tag{4}$$