Physica A 401 (2014) 182–200

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Pricing of range accrual swap in the quantum finance Libor Market Model

Belal E. Baaquie [a,b], Xin Du [a,*], Pan Tang [c], Yang Cao [a]

[a] Department of Physics, National University of Singapore, 2 Science Drive 3, Singapore 117542, Singapore
[b] Risk Management Institute, National University of Singapore, Singapore
[c] Department of Finance, School of Economics and Management, Southeast University, Nanjing, Jiangsu 210096, China

HIGHLIGHTS

- The range accrual swap is modelled in the framework of Quantum Finance and the approximate price is obtained using an expansion in the Libor volatility.
- The price of accrual swap is numerically analysed by generating daily sample values of a two dimension Gaussian quantum field.
- The Monte Carlo simulation method is used to study the nonlinear domain of the model and determine the range of validity of the approximate formula.

ARTICLE INFO

Article history:
Received 27 September 2013
Received in revised form 12 November 2013
Available online 28 January 2014

Keywords:
Quantum finance
Range accrual swap
Monte Carlo simulation

ABSTRACT

We study the range accrual swap in the quantum finance formulation of the Libor Market Model (LMM). It is shown that the formulation can exactly price the path dependent instrument. An approximate price is obtained as an expansion in the volatility of Libor. The Monte Carlo simulation method is used to study the nonlinear domain of the model and determine the range of validity of the approximate formula. The price of accrual swap is analyzed by generating daily sample values by simulating a two dimension Gaussian quantum field.

1. Introduction

Range accrual swap is a type of a derivative product that is similar to normal interest rate swap. The investor may have a view that the market is or is not very volatile and that consequently some index will or will not stay within a predefined range. The range accrual index could be an interest rate, an FX rate or a commodity price. The investor makes an additional profit if the view taken is correct and loses money otherwise. The range accrual can also serve to hedge risks since the payments are based on daily observations and not on a pre-fixed rate.

The interest range accrual swap is one of the most popular non-vanilla interest rate derivatives; more than USD 160 billion of range accrual indexed on interest rates have been sold since 2004, and the total volumes have been increasing rapidly in the last few years. The present work investigates the range accrual swap based on the behavior of the 3-month Libor.

The range accrual for interest rates has been studied in many books and articles, such as Navatte and Quittard-Pinon [1], Nunes [2] using the Gaussian HJM (Heath–Jarrow–Morton) framework, Damiano Brigo and Fabio Mercurio using the BGM-

* Corresponding author. Tel.: +65 96458390.
E-mail addresses: phybeb@nus.edu.sg (B.E. Baaquie), duxin.nus@gmail.com (X. Du), pantangnus@gmail.com (P. Tang), yangcao@nus.edu.sg (Y. Cao).

0378-4371/$ – see front matter Crown Copyright © 2014 Published by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.physa.2014.01.042

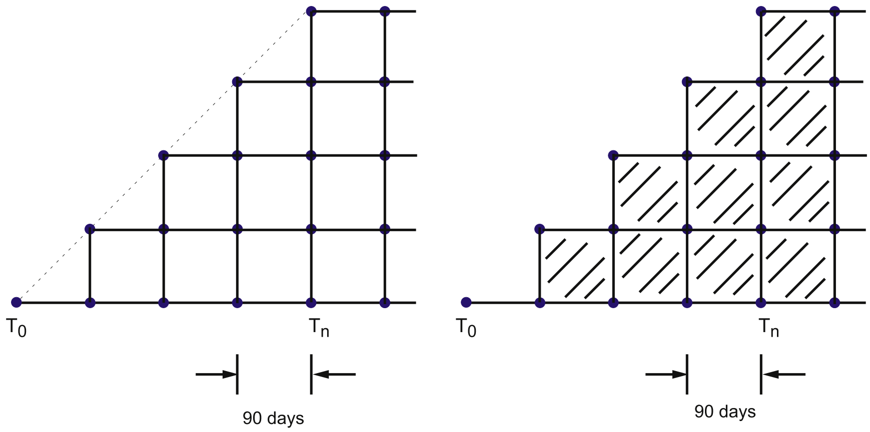

Fig. 1. The Libor lattice defined by $L(T_n, T_m)$. The shaded portion shows the time dependence of the Libor rates for the Libor lattice.

Jamshidian formulation of LMM [3] and Yoon and Jang [4] with jump risk. The quantum finance formulation of the Libor Market Model has been studied in Ref. [5].

Quantum field theory has been applied to many *classical problems*; two famous examples are (a) the solution of classical phase transitions by Wilson and which led to his physics Nobel Prize in 1982 [6] and (b) the complete classification of knots and links in three dimensions by Witten, for which he was awarded the Fields Medal in 1989 [7]. These examples show that the mathematics of quantum mechanics extends far beyond not only quantum systems but instead can be applied to a wide variety of phenomena, including even the social sciences [8]. The formalism of quantum finance has been developed in this spirit and is based on the application of *quantum mathematics* to finance [9,10]. In particular, interest rate models in quantum finance are based on the mathematics of a two-dimensional Euclidean quantum field.

This paper is organized as follows. Section 2 briefly reviews LMM in quantum finance. In Sections 3 and 4 the range accrual swap is defined in the mathematical framework of the quantum LMM. In Section 5 we derive an analytical approximate formula for the price of the range accrual swap. In Sections 6, 7, we carry out a Monte Carlo simulation to evaluate the price of the range accrual swap. In particular, we compare the approximate analytical expression for the range accrual swap with simulation results. We draw some conclusions in the final Section 8.

2. Libor market model

The forward interest rate, denoted by $f(t, x)$, is the interest rate fixed at time t for an overnight loan at future time $x > t$. Both the bond market and interest paid on cash deposits are determined by $f(t, x)$. The standard industry bench mark is given by two models, namely the HJM model that is used to price bonds and the BGM-Jamshidian model that quantifies the interest rate paid on cash deposits.

To mathematically define the industry bench-mark models let $R(t)$ be Gaussian white noise with correlators given by

$$E[R(t)] = 0; \qquad E[R(t)R(t')] = \delta(t - t').$$

The HJM model is a linear model defined by

$$\frac{\partial f(t, x)}{\partial t} = \alpha(t, x) + \sigma(t, x)R(t) \tag{1}$$

where $\alpha(t, x)$, $\sigma(t, x)$ are deterministic functions.

The Libor interest rate $L(t, T_n)$ is a simple interest rate, fixed at time t, for making a cash deposit at future time T_n for a duration of time ℓ, called the tenor of the deposit. Simple interest Libor rates $L(t, T_n)$ are defined on the Libor future time lattice defined by $T_n = n\ell$, where ℓ is Libor tenor. The Libor lattice is shown in Fig. 1; for fixed tenor ℓ Libor rates are only defined on the Libor interval $[T_n, T_{n+1}]$; the shaded portion in Fig. 1 shows all the Libor rates that exist for the Libor lattice.

In terms of Libor forward interest rates $L(t, T_n)$ is given by

$$1 + \ell L(t, T_n) = e^{\int_{T_n}^{T_{n+1}} dx f(t,x)}. \tag{2}$$

BGM-Jamshidian formulation of LMM is defined by

$$\frac{1}{L(t, T_n)} \frac{\partial L(t, T_n)}{\partial t} = \xi_n(t) + \gamma_n(t)R(t) \tag{3}$$

where $\gamma_n(t)$ is a deterministic function; $\xi_n(t)$ is a function of Libor rates $L(t, T_n)$ and hence makes the model nonlinear.