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Can log-periodic power law structures arise from random fluctuations?

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HIGHLIGHTS

- We estimate the stochastic process of DAX log-returns.
- The resulting Langevin equation is used to run extensive simulations.
- Our findings indicate that this equation is capable of generating LPPL structures.
- A link between synthetic LPPL patterns and ensuing phase transitions is established.
- We statistically confirm the genuine nature of LPPL structures in the DAX.

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ABSTRACT

Recent research has established log-periodic power law (LPPL) patterns prior to the detonation of the German stock index (DAX) bubble in 1998. The purpose of this article is to explore whether a Langevin equation extracted from real world data can generate synthetic time series with comparable LPPL structures. To this end, we first estimate the stochastic process underlying the DAX log-returns during the period from mid-1997 until end-2003. The employed data set contains about $3.93 \cdot 10^6$ intraday DAX quotes at a sampling rate of 15 s. Our results indicate that the DAX log-returns can be described as a Markov process. As a consequence, a Langevin equation is derived. Based on this model equation, we run extensive simulations in order to generate 100 synthetic DAX trajectories each covering 3000 trading days. We find LPPL behavior in ten artificial time series. Moreover, we can establish a link between LPPL patterns and ensuing bubble bursts in seven synthetic 600-week windows. However, the LPPL components in most synthetic trajectories differ fundamentally from those LPPL structures that have previously been detected in real financial time series. Summarized, this paper demonstrates that LPPL structures are not necessarily the signature of imitative behavior among investors but can also stem from noise, even though the likelihood of this is extremely low. Thus, our findings confirm with high statistical confidence that the LPPL structures in the DAX development are rooted deeper than only in the random fluctuations of the German stock market.

1. Introduction

Fluctuations and correlations play an important role in both thermodynamic phase transitions and turbulences on financial markets [1]. The nonlinear properties of complex financial systems have attracted the research interests of many physicists in recent years. As a consequence, econophysics developed into an interdisciplinary field of research. Since then, an extensive literature employing physical methods on financial issues has evolved. Two lines of research within econophysics

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are of particular importance for this paper. The first research direction presents an approach to model complex financial systems in the form of a Langevin equation. The second strand of research explores the hypothesis that log-periodic power law (LPPL) structures in financial time series are predictors of speculative bubble bursts. Here, these two fields of research are merged in order to investigate whether the LPPL patterns guiding the inflation of the German stock index (DAX) until 1998 [2] are genuine rather than spurious.

As a consequence of self organization, the behavior of complex systems can often be traced back to simple model equations [3]. A promising approach to describe such systems is via Langevin equations which may involve nonlinearities in both the deterministic and the stochastic part [4]. Siegert et al. proposed a general data-driven approach for extracting such model equations directly from experimental data [5]. This approach has already been quantitatively validated by applying it to a wide variety of real world data sets. Examples include tremor data of patients suffering from Parkinson's disease [6], data of a chaotic electric circuit [6], and the dynamics of traffic flow [7]. Furthermore, this approach has proven to be appropriate for modeling the fluctuations on stock markets [8,9] and of daily oil prices [10], as well as for analyzing the analogy between turbulence and US Dollar–German Mark exchange rates [11–13].

Log-periodicity – a concept originating from statistical physics – has been demonstrated to be valuable for forecasting collapses of speculative bubbles in financial markets. In a nutshell, the log-periodic hypothesis contends that speculative bubbles are characterized by a faster-than-exponential power law growth decorated by log-periodic oscillations. Thus, the LPPL provides an opportunity to quantify the speculative aspects of financial markets. The detection of log-periodic precursors in noisy financial data before crashes goes back to the pioneering works of Feigenbaum and Freund as well as of Sornette et al. who – inspired by the apparent analogy between financial crashes and phase transitions in complex physical systems – detected ex-post LPPL structures guiding the S&P500 index prior to the crash in October 1987 independently from each other [14,15]. Since then, five strands of literature have developed. First, a vast body of literature accumulated establishing LPPL structures in a wide variety of financial markets before crashes which extend over periods from a few months [16] to several years [17]. For example, log-periodic structures were discovered in numerous stock market bubbles [1,18–20], in the 2006–2008 oil bubble [21], in the US FED Prime Rate [22], and in real estate bubbles [23,24]. Another testing ground for the LPPL have been time series related to credit risk [25–29]. Chang and Feigenbaum summarize all these case studies by stating that "log-periodic precursors have been identified before most and perhaps all financial crashes of the Twentieth Century" [30]. Second, some authors mathematically formalized the behavior of market participants in order to theoretically found the observed log-periodic patterns prior to crashes [31,32]. For example, Johansen et al. proposed an Ising Spin model to explain how the imitation between investors is leading to observable signatures in the form of LPPLs [32,33]. Third, there is rich literature demonstrating that the herding behavior of investors not only results in speculative bubbles with accelerating market overvaluations, but also in anti-bubbles with decelerating market devaluations [34–36]. A fourth stream of literature goes even one step further by claiming that financial crashes and rebounds can be forecasted by employing the LPPL [37]. The central idea of this research direction is the integration of LPPL parameters into pattern recognition approaches in order to estimate end times of bubbles and anti-bubbles [36–39]. However, the hypothesis that the log-periodic concept possesses predictive power has met skepticism and even triggered a heated discussion between three discoverers [17,40,41].

Due to these evidences, the question arises whether the detected log-periodic signals are genuine or spurious, i.e., whether LPPL structures are indeed the result of imitative behavior among investors or whether these structures can be ascribed to random fluctuations. Chang and Feigenbaum sum up: "It remains to be established that [log-periodic] fluctuations actually reflect the structure of financial markets and do not arise by chance" [30]. Therefore, the fifth branch of research investigates whether stochastic processes are capable of generating log-periodic patterns and, if so, whether there is a connection between LPPL structures and impending crashes in synthetic data. To the best of our knowledge, only two papers on this topic have been published so far [14,32]. Feigenbaum and Freund modeled the stock market trend as a random walk governed by a probability distribution of daily index changes which is composed of three Gaussian distributions [14]. They have detected 17 periods of LPPL structures in a synthetic time series of 10,000 days. This number is fairly large compared to three periods of LPPL patterns in real S&P500 data over 3800 days. However, they have discovered that the presence of LPPL structures is only predictive of crashes in real data. A connection between LPPL structures and crashes in the synthetic data could not be established. Johansen et al. generated 1000 synthetic data sets of a length of 400 weeks by means of a GARCH(1,1) process in order to explore whether this stochastic process can explain the presence of LPPL patterns [32]. They only found LPPL structures in two 400-week windows which corresponds to a confidence level of 99.8% for rejecting the hypothesis that a GARCH(1,1) process can generate LPPL structures. They also could not establish a causal link between log-periodic patterns and crashes. Thus, they conclude "that real markets exhibit behaviors that are dramatically different from the one predicted by a GARCH(1,1) process" [41].

Feigenbaum is right to state that "all of these simulation results are weakened by the fact that each experiment rules out only one possible data-generating process" [17]. However, Sornette and Johansen reply that "we shall never be able to 'prove' in an absolute sense the existence of a log-periodicity genuinely associated with specific market mechanisms. The next best thing we can do is to take one by one the best benchmarks of the industry and test them to see if they can generate the same structures as we document" [41]. Following these remarks, we revisit the LPPL structures in the evolution of the DAX from January 1995 until March 2000 [2,16]. We investigate whether the underlying stochastic process of the DAX log-returns modeled by a Langevin equation can generate LPPL structures and, if so, whether there is a link between log-periodic oscillations and succeeding drawdowns in the synthetic data. In doing so, we statistically examine whether log-periodic