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On microscopic theory of spin-S Bose–Einstein condensate in a magnetic field

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Abstract

The ground state and low-lying collective modes of Bose–Einstein condensate (BEC) of atoms with arbitrary spin in a magnetic field is studied using Bogoliubov's model for weakly interacting Bose gas. The equation for vectorial order parameter valid at temperature $T \to 0$ is derived and its specific solution is found. This solution corresponds to the formation of BEC of atoms with a definite spin projection onto direction of magnetic field. We study the thermodynamic stability of the found ground state and obtain the expressions for low-lying collective modes.

Keywords: Bose–Einstein condensation; Spinor condensate; Weakly interacting Bose gas; Quasiaverages; Thermodynamics; Excitation spectra

1. Introduction

After the first remarkable experiments concerning the observation of BEC in dilute gases of alkali atoms such as ^{87}Rb [1], ^{23}Na [2], and ^{7}Li [3] the interest to this phenomenon has revived [4,5]. Later, BEC has been also obtained in other atomic species: atomic hydrogen [6], metastable ^{4}He [7], and ^{41}K [8]. The experimental realization of BEC has become possible due to development of laser cooling and trapping techniques [9]. The carried out experiments have proved many predictions of the microscopic theory for weakly interacting Bose gas, which originates from the pioneering work of Bogoliubov [10]. Bogoliubov's theory has become almost the first theory in which it was necessary to move essentially from the methods of standard perturbative approach while describing the interaction effects. However, this theory, in its original formulation, did not take into account the internal degrees of freedom of atoms. The effect of spin degrees of freedom for weakly interacting Bose gas (spinor BEC) has been studied in Refs. [11–19].

The realization of optical trapping for atomic condensate [20] has stimulated theoretical interest to spinor BEC. Bose condensation in a weakly interacting gas of bosonic atoms has been studied theoretically by many

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authors both for spin-1 [12–17] and spin-2 [18,19] bosons. These investigations are based on the effective interaction Hamiltonians of two bosons in which the interaction is characterized by a definite number of interaction constants—*s*-wave scattering lengths. The number of scattering lengths is determined by the total spin of two interacting bosons taking into account the symmetry properties of their wave function. For example, in case of spin-1 atoms the interaction Hamiltonian contains two interaction constants [12–17], in case of spin-2 atoms there are three interaction constants [18,19]. Thus, as the spin value of atoms grows, the number of constants, which characterize the interaction of two bosons is increased. Note that in the mentioned effective Hamiltonians it is difficult to interpret the physical nature of the separated term of non-relativistic interaction not associated with neither potential nor spin-exchange interactions (see e.g. Ref. [18]).

In this paper we study a weakly interacting Bose gas of particles with arbitrary integer spin S in a magnetic field (see also Ref. [11]). We start from the microscopic interaction Hamiltonian for two spin-S bosons. This Hamiltonian is specified by two functions, which describe potential and spin-exchange interactions of spin-S atoms. According to general rules of quantum mechanics we pass from the pairwise interaction of two bosons to the standard expression for binary interaction of arbitrary number of bosons in the second quantization representation. By solving the multichannel scattering problem for the considered Hamiltonian we could find, in principle, all scattering lengths in terms of the functions characterizing the potential and spin-exchange interactions. Thereby, it would be possible to obtain the Hamiltonians analogous to the above mentioned effective interaction Hamiltonians (see e.g. Ref. [18]). However, the use of the microscopic Hamiltonian gives a possibility to restrict ourselves by two interaction constants even in the case of arbitrary spin when studying the ground state, stability, and excitations in a weakly interacting gas in the presence of BEC.

2. Method of quasiaverages and the model with a separated condensate

To describe the system with a spontaneously broken symmetry we address to the method of quasiaverages [21,22]. According to this method the Gibbs statistical operator is modified so that it possesses the symmetry of degenerate state. This modification is usually done by introducing the infinitesimal "source" $\nu\hat{F}$ ($\nu \to 0$) into the Gibbs exponent, which has the symmetry of phase under consideration. Then, the average value of any physical quantity $\hat{A}$ is defined as

$$\prec \hat{A} \succ = \lim_{\nu\to 0}\lim_{V\to\infty} \mathrm{Tr}\,\hat{w}_\nu \hat{A}, \tag{1}$$

where the Gibbs statistical operator $\hat{w}_\nu$ has the form

$$\hat{w}_\nu = \exp(\Omega_\nu - \beta(\hat{H} - \mu\hat{N} + \nu\hat{F})). \tag{2}$$

Here $\beta = 1/T$, μ are the reciprocal temperature and chemical potential, respectively, and $\hat{H}$, $\hat{N}$ are the system Hamiltonian and the particle number operator. The thermodynamic potential Ω_ν being a function of thermodynamic parameters β, μ is found from the normalization condition $\mathrm{Tr}\,\hat{w}_\nu = 1$. Notice that the limits in (1) are not permutable.

Consider a gas of condensed bosonic atoms with spin S. The formation of a condensate is accompanied by the gauge symmetry breaking and, therefore, in order to remove this kind of degeneracy we should choose the "source" $\nu\hat{F}$ in (2) such that $[\hat{w}_\nu, \hat{N}] \neq 0$ ($\hat{N}$ is a generator of phase transformation),

$$\nu\hat{F} = \nu_\alpha \int \mathrm{d}^3x(\hat{\psi}^\dagger_\alpha(\mathbf{x}) + \hat{\psi}_\alpha(\mathbf{x})), \tag{3}$$

where $\hat{\psi}^\dagger_\alpha(\mathbf{x})$, $\hat{\psi}_\alpha(\mathbf{x})$ are the creation and annihilation operators with index α taking $2S+1$ values (the summation over repeated indices is assumed). Then according to (1), (2) $\prec \hat{\psi}_\alpha(\mathbf{x}) \succ = V^{-1/2} \prec \hat{a}_{0\alpha} \succ \sim 1$ that corresponds to the formation of atomic condensate with momenta $\mathbf{p} = 0$. The order parameter $\Psi_\alpha = V^{-1/2} \prec \hat{a}_{0\alpha} \succ$ is called the condensate wave function.

The method of quasiaverages and the spatial correlation decay principle one allow to justify the replacement of creation and annihilation operators of atoms with momentum $\mathbf{p} = 0$ by c-numbers, $\hat{a}_{0\alpha}, \hat{a}_{0\alpha}{}^\dagger \to \sqrt{V}\Psi_\alpha, \sqrt{V}\Psi^*_\alpha$ [21–23] (the condensate separation procedure).