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Mixed spin-2 and spin-$\frac{5}{2}$ Blume–Emery–Griffiths model

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Abstract

The mixed spin-2 and spin-$\frac{5}{2}$ Blume–Emery–Griffiths (BEG) Ising ferrimagnetic system is studied on the Bethe lattice using the exact recursion equations with the coordination number $q = 3$ corresponding to the honeycomb lattice on real lattices. The influences of the crystal field and the biquadratic interactions are investigated by obtaining the phase diagrams on the $(K/|J|, kT/|J|)$ and $(D/|J|, kT/|J|)$ planes, respectively, with equal crystal field interactions for the sublattices. The model presents very rich critical behaviors, which includes the first- and second-order phase transitions, thus also the tricritical and critical end points are observed. We have also found that the model gives up to four compensation temperatures for appropriate values of the system parameters.

Keywords: Mixed spin; Bethe lattice; Recursion relations; Spin-2; Spin-$\frac{5}{2}$

1. Introduction

The molecular-based magnetic materials with spontaneous magnetic moments are of great interest for their potential applications such as in thermomagnetic recording and in devices [1] and it is believed that ferrimagnetic ordering plays a crucial role in some of these materials. Therefore, the synthesis of new ferrimagnetic materials is an active field in material science.

The ferrimagnetic materials consist of two unequal magnetic moments, i.e. a bipartite lattice with spin-σ_A and spin-σ_B with $\sigma_A \neq \sigma_B$, which interact antiferromagnetically, therefore, their moments do not cancel each other at low temperatures except at the compensation temperatures. The existence of compensation temperatures in ferrimagnets has interesting applications such as the magneto-optical recording. The compensation temperature is the temperature at which the net magnetization, i.e. the difference of the sublattice magnetizations, vanish before the critical temperature [2].

The Ising ferrimagnetic systems, $J < 0$, with many combinations of unequal magnetic moments have been studied with exact or many approximate methods. It is well known that as the spin values gets higher for a magnetic system, i.e. for high spins, the problem gets more difficult but the phase diagrams become much richer. Therefore, in this work we consider two such magnetic moments with high spin values, spin-2 and spin-$\frac{5}{2}$, hoping that we can obtain very rich and interesting critical behaviors in the phase diagrams.

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This ferrimagnetic mixed-spin system with spin-2 and spin-$\frac{5}{2}$ was already studied by using a few different techniques such as: The existence of the compensation temperatures on a layered honeycomb lattice [3] and on a two-dimensional honeycomb lattice were carried out with a Monte Carlo (MC) simulation [4]. The compensation points on the honeycomb lattice layers which are coupled with two kinds of positive interlayer coupling constant [5] and its internal energy, specific heat and the initial susceptibility [6], the magnetic properties of the ferrimagnetic system with dilution [7] and also including the effect of the transverse field [8] were all investigated on the basis of the effective-field theory (EFT) with correlations. The magnetic properties of the Heisenberg ferrimagnetic system on a layered honeycomb lattice was examined by using the multisublattice Green-function technique which takes into the quantum nature of Heisenberg spins [9] and its compensation behavior [10] and tricritical points of a layer Ising system [11]. The low-temperature properties were studied on the honeycomb lattice using a linear spin–wave theory [12]. The final work that we can report is the Bethe lattice (BL) consideration of the mixed spin-2 and spin-$\frac{5}{2}$ Blume–Capel model with two different crystal fields for the sublattices by the use of the exact recursion relations [13]. It should be emphasized that all the works mentioned above does not include the biquadratic exchange interaction, which means that in these works only the Blume–Capel model was considered.

Hence, we study the mixed spin-2 and spin-$\frac{5}{2}$ Blume–Emery–Griffiths (BEG) model Ising ferrimagnetic system on the BL in terms of exact recursion relations to investigate the influence of the biquadratic and crystal field interactions on the critical behaviors for $q = 3$ with equal crystal field interactions for the sublattices. At this point, we should mention about the BL, i.e. it is an infinitely Cayley or regular tree, i.e. a connected graph without circuits, and historically gets its name from the fact that its partition function is exactly that of an Ising model in the Bethe approximation [14]. The importance is that the BL is an infinitely tree gives us the negligible boundary effects, therefore, far from the boundary sites that is deep inside the Cayley tree, now BL, all the sites become equivalent, thus studying the behavior of one spin, named as the central spin, is enough to obtain the full picture of the system. Thus, we should also note that the choice of the central-spin does not change the behavior of the system [15]. The BL calculations provides exact solutions and results of which qualitatively better approximations for the regular lattices than solutions obtained by the conventional mean-field theories [16]. In addition, the cluster variation method in the pair approximation studies on regular lattices yield results that are exact for the same model on the BL [17]. Of course, the BL considerations also have some limitations that is it predicts a transition temperature higher than that for a regular lattice and it is not reliable for predicting critical exponents [18], where also the correspondence of the BL with regular lattices and real physical systems and whether it can be embedded into a finite-dimensional Euclidean space are also discussed. Therefore, in this work we have employed the use of the BL in terms of the recursion relations [19] obtained for the single spin systems and improved for the mixed spin systems [15] to study the mixed spin-2 and spin-$\frac{5}{2}$ ferrimagnetic system.

The rest of this paper is constructed as follows. In the next Section, the model on the BL is discussed and then the exact expressions for the order-parameters, dipolar and the quadrupolar moments, are obtained in terms of the recursion relations. The Section 3 is devoted to finding of the critical temperatures, i.e. the Curie or the second-order phase transition temperature, the free energy, which is used to find the first-order phase transition temperatures, and the definition of the compensation temperature. In the last section, the phase diagrams in the $(K/|J|, kT/|J|)$ and $(D/|J|, kT/|J|)$ planes for given values of $D/|J|$ and $K/|J|$ are discussed in detail, respectively, where also at the end we give a brief summary and discussion.

2. The formulation in terms of the recursion relations on the BL

The Hamiltonian of the mixed spin-2 and spin-$\frac{5}{2}$ BEG ferrimagnetic system in terms of the bilinear (J), biquadratic (K) and crystal field interactions (D) is given by

$$\mathscr{H} = -J\sum_{\langle i,j\rangle}\sigma_i S_j - K\sum_{\langle i,j\rangle}\sigma_i^2 S_j^2 - D\left[\sum_i \sigma_i^2 + \sum_j S_j^2\right], \tag{1}$$

where each σ_i located at site i is a spin-2 and can take the values ± 2, ± 1, 0 and each S_j located at site j is a spin-$\frac{5}{2}$ and can take the values $\pm\frac{5}{2}$, $\pm\frac{3}{2}$, $\pm\frac{1}{2}$. In the case of mixed spins, the BL is set up in such a way that the