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Correlation functions of an autonomous stochastic system with time delays

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HIGHLIGHTS

- Correlations of the autonomous stochastic delay system are investigated.
- The delay time makes correlations of the system oscillate and attenuate periodically.
- The oscillation period of correlations increases with the delay time.
- The delay time enhances the auto-correlation and the cross-correlation.
- The noise strength lowers the auto-correlation and the cross-correlation.

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ABSTRACT

The auto-correlation function and the cross-correlation function of an autonomous stochastic system with time delays are investigated. We obtain the distribution curves of the auto-correlation function $C_x(s)$ and $C_y(s)$, and the cross-correlation function $C_{xy}(s)$ and $C_{yx}(s)$ of the stochastic dynamic variables by the stochastic simulation method. The delay time changes prominently the behaviors of the dynamical variables of an autonomous stochastic system, which makes the auto-correlation and the cross-correlation of the autonomous stochastic system alternate oscillate periodically from positive to negative, or from negative to positive, decrease gradually, and finally tends to zero with the decay time. The delay time and the noise strength have important impacts for the auto-correlation and the cross-correlation of the autonomous stochastic delay system. The delay time enhances the auto-correlation and the cross-correlation, on the contrary, the noise strength lowers the auto-correlation and the cross-correlation. Under the time delay, by comparison we further show differences of the auto-correlation and the cross-correlation between the dynamical variables x and y.

1. Introduction

In a stochastic dynamics process, the correlation function is a very important quantity describing the dynamical behavior of stochastic variables with time, and therefore the analysis of steady-state correlation functions is a very useful tool in the study of a wide range of a nonlinear stochastic system [1,2]. Over the last decades, the research of the correlation function of the stochastic system has attracted a great deal of attentions, where many interesting and important results have been given in a series of experimental and theoretical papers [3–22]. The correlation function characterizes the related activity of the

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dynamic variables between two states at different time. The correlation function can be classified into the auto-correlation function which shows the related activity of the same variable at different time and the cross-correlation function which shows the related activity between different variables at different time.

A nonlinear autonomous system of a two-dimensional model possesses many important properties, such as characteristics of the phase line, periodicity, and aperiodicity. By discussing the dynamic properties of an autonomous system driven by noise, many interesting results can be obtained in applications of physics, chemistry, biology, and other fields, such as the forced pendulum, the Duffing equation, the Van der Pol oscillator, and so on [23–32]. Investigating the auto-correlation function and the cross-correlation function of dynamics variables of an nonlinear autonomous system with noise, we can further understand the dynamic properties of the nonlinear autonomous stochastic system.

In the natural world, many realistic complex systems can be regarded as to obey self-regulating control, affect, and optimize mechanisms in terms of feedback loops where the output signal or other key quantities are fed to the input interfaces, in particular, which can make the optimal and effective output in the stochastic system driver by noise [33–41]. In the meanwhile, usually, the transmission takes some time, that is to say, the input signals are related to the output signals at earlier time, and the effect of feedback of the output to the input possesses the time delay [42–48], including population systems, neural network systems, motor control systems, laser systems, chemical reaction systems, and recycled noise systems [49–64]. So investigating the dynamic properties of the practical stochastic systems, we should take into account the effects of time-delayed feedback. In recent years, many significant studying results on stochastic systems with time-delayed feedback have been obtained. In a fully synchronized state the time delay induces the clustering, and the multistability [65]. The Experimental phenomenon of time-delay-induced amplitude death in two coupled nonlinear electronic circuits are observed [66]. The time delay may generate the perfect synchrony in the Kuramoto model [67] and the phenomenon of slow switching in globally coupled oscillators [68]. In the stochastic system with an external driving force, the time delay induce positive motion of the particle [69], and restrains the unbounded reproducibility of species [70]. For the stochastic system without an external signal, through interaction of noise with delay time, the regularity of the oscillatory states is maximal for a certain noise level [71], and interesting coherence resonance is induced by the time delay [72].

This paper is organized as follows: In Section 2, we introduce the auto-correlation function and the cross-correlation function of the dynamics variables of a two-dimension stochastic system, and present the auto-correlation and the cross-correlation of the nonlinear autonomous stochastic system without the time delay. In Section 3, we present the auto-correlation function and the cross-correlation function of the nonlinear autonomous stochastic system with time delays, discussing the effects of the delay time. Finally, summary and conclusions conclude the paper in Section 4.

2. The auto-correlation and the cross-correlation of a two-dimension autonomous stochastic system

2.1. Auto-correlation functions and cross-correlation functions of a two-dimension model

For a general stochastic process of a two-dimensional model for which a stationary state exists, the stationary auto-correlation functions of the dynamic variables X_i $(i = 1, 2)$ are defined as [1,10]

$$C_{X_i}(s) = \langle \delta X_i(t+s)\delta X_i(t)\rangle_{st} = \lim_{t\to\infty} \langle \delta X_i(t+s)\delta X_i(t)\rangle, \tag{1}$$

where

$$\delta X_i(t+s) = X_i(t+s) - \langle X_i(t+s)\rangle, \tag{2}$$

and

$$\delta X_i(t) = X_i(t) - \langle X_i(t)\rangle. \tag{3}$$

The normalized auto-correlation functions are given by

$$C_{X_i}(s) = \frac{\langle \delta X_i(t+s)\delta X_i(t)\rangle_{st}}{\langle (\delta X_i(t))^2\rangle_{st}} = \lim_{t\to\infty} \frac{\langle X_i(t+s)X_i(t')\rangle - \langle X_i(t)\rangle^2}{\langle (\delta X_i(t))^2\rangle}. \tag{4}$$

The stationary normalized cross-correlation functions between dynamic variables X_i and X_j $(i, j = 1, 2$ and $i \neq j)$ are given by [73]

$$C_{X_iX_j}(s) = \frac{\langle \delta X_i(t+s)\delta X_j(t)\rangle_{st}}{\sqrt{\langle (\delta X_i(t+s))^2\rangle_{st}}\sqrt{\langle (\delta X_j(t))^2\rangle_{st}}} = \lim_{t\to\infty} \frac{\langle \delta X_i(t+s)\delta X_j(t)\rangle}{\sqrt{\langle (\delta X_i(t+s))^2\rangle}\sqrt{\langle (\delta X_j(t))^2\rangle}}. \tag{5}$$

2.2. The auto-correlation and the cross-correlation of a two-dimension autonomous stochastic system

An autonomous system of a two-dimensional model which can possess a limit cycle, two fixed points, or two stable fixed points and two unstable fixed points, these behaviors of which can be generate or eliminated by adjusting the control