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Bose–Einstein condensation of a two-dimensional harmonically trapped q-deformed boson system

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HIGHLIGHTS

- We explore BEC of a q boson system with finite particles in a 2D harmonic trap.
- We obtain the expression for the effective critical temperature T_c among others.
- We compare the results of our research with the experimental data.

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ABSTRACT

Within the framework of the theory of q-deformed bosons, Bose–Einstein condensation of a q-deformed boson system, which has a finite number of particles and is trapped in a two-dimensional harmonic potential trap, is investigated. By using the approach of modified density of states, we obtain the expressions of the effective critical temperature T_c, the ground state population N_0/N and the specific heat C. Further numerical calculation shows that Bose–Einstein condensation of such a system has some particular and interesting features, such as it is possible to well fit the experimental data concerned by adjusting the parameter q, and so perhaps we may treat the actual bosons as q-deformed bosons to some extent. We consider that this work may provide much insight into the theory of q-deformed bosons, and may also be useful for the further study on Bose–Einstein condensation of a concrete two-dimensional trapped q-deformed boson system.

1. Introduction

Seventy-one years after the prediction of Bose and Einstein [1] about the phenomenon referred to as Bose–Einstein condensation (BEC), the observation of BEC was first accomplished in dilute gases of atomic rubidium [2] and lithium [3] in 1995. In the two experiments above the atoms are confined in harmonic-oscillator potentials, and the exciting experimental results thus set off a wave of investigation on BEC in harmonic potential traps that has continued to the present time [4–8]. In the literature above, several research methods, which ignore the interactions between atoms, are adopted for the purpose of providing more reliable and accurate explanation for the experimental results. For instance, Grossmann and Holthaus [4] postulated an expression of the modified density of states (MDOS) and gave beautiful analysis on BEC of a boson system in a harmonic potential trap. Later on, taking advantage of the contour integration, Kirsten and Toms [5] introduced one more accurate method and their theory can recover the results of [4] in a reasonable and precise manner. Almost simultaneously, by applying the high-temperature expansion, Ketterle and Druten [6] found another accurate method and obtained the same results as those of [4]. Additionally, several authors have developed other kinds of new approaches [9–15], which take into account the effects of finite size of a system or the actions between particles, to analyze BEC of a trapped boson system

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and have obtained excellent results, which are mostly in good agreement with the measured results reported in another two important experiments [16]. For example, in [9], the authors investigated a trapped Bose gas interacting with repulsive forces and obtained the temperature dependence of various thermodynamic quantities, by using a mean-field approach and applying the Popov approximation or the WKB semi-classical approximation. In [10], the Bose condensed gas is successfully described by a semi-ideal two-gas model. In [15], by introducing a correction term, which may result from the interatomic interaction, to the MDOS, the authors gave good predicted results for the condensed fraction and the critical points.

Along with these investigations on BEC of a three-dimensional (3D) trapped Bose gas, analogous research about BEC of a two-dimensional (2D) trapped Bose gas has also attracted considerable interest [17–30]. Although it is well known that a uniform ideal or interacting 2D Bose gas does not undergo BEC at finite critical temperature [27,31], a homogeneous 2D Bose gas with repulsive interactions between particles may experience a phase transition from a normal state to a superfluid state. This particular phase transition was first predicted by Berezinskii, Kosterlitz and Thouless (BKT) [32], and now it has been observed in macroscopic quantum systems [17,23]. Recently, several experiments [25–27,29] about a quasi two-dimensional (Q2D) trapped Bose gas have been carried out, and it is found that for an actual Q2D trapped Bose gas both the BKT and the BEC transition may occur at finite critical temperature. To be more exact, for such a Bose gas the BKT transition is more apt to take place than the BEC transition, and the latter can be viewed as a special, noninteracting limit of the more general BKT behavior. For instance, in the experiments performed by Krüger et al. [25] and Hadzibabic et al. [27], a gas of rubidium atoms is trapped by using a combination of a magnetic trap providing harmonic confinement in the xy-plane, and an optical lattice, ensuring that the third degree of freedom (z) of the gas was frozen. The results of the experiments show that a well-defined critical point may be identified, which separates a high temperature phase with a single component density distribution, and a low temperature phase with a clear bimodal distribution. These results also demonstrate that such a Q2D trapped Bose gas cannot be regarded as an ideal 2D trapped Bose gas. In order to give a reasonable explanation for the experimental results above, several theoretical approaches have been proposed in the past decade [27,28,30]. Among these theories, the quantum Monte Carlo (QMC) analysis and the mean field (MF) approach are more reliable and successful. In [28], utilizing the semi-classical approach and the QMC analysis, the authors computed the density profile and the condensate fraction of an interacting Q2D trapped Bose gas. Moreover, their methods may provide a good result for the experiment of Krüger et al. [25] by using the effective temperatures. In Ref. [27], the authors predicted a strong correction to the critical atom number with respect to the ideal gas theory, by employing the local density approximation, the MF theory and the QMC analysis. It is also found that quantitative agreement between theory and experiment may be reached concerning the critical atom number providing the predicted density profiles are used for temperature calibration. So far, we notice that the discussions above are all about the BEC of standard bosons, and therefore a natural question is: what would happen to the phenomenon of BEC of q-deformed bosons (hereinafter referred to as q bosons). Fortunately, it has been demonstrated that free q bosons may undergo BEC under appropriate conditions [33,34], and BEC of a 3D trapped q boson system with a finite number of particles has also been investigated [35]. However, as far as we know, BEC of a 2D trapped q boson system has not been discussed as yet, and thus in the present paper, we try to explore this problem referring to the case of standard bosons. Moreover, it has been suggested that BEC even could occur for excitons [36,37] which may be treated as q bosons under a certain condition [38], and so we consider that our work has the following significance: (a) the present paper may be a development of the theory of q bosons, and (b) in experiments on BEC of bosons, perhaps it is more suitable and reasonable to treat these microscopic particles as q bosons rather than standard bosons, because of the interactions between particles and the finite size of the system.

The theory of q bosons is one theory of the intermediate statistics, which is somewhat different from the standard quantum statistical mechanics, and originated from the study on exactly solvable statistical systems, which has led to the q-deformed algebra of creation and annihilation. This theory provides a new quantum statistics of many-body systems and has a wide range of applications envisaged – from black holes [39] to nuclear physics [40], etc. The crucial idea of this theory is to deform the quantum algebra of the creation and annihilation operators of bosons, thus modifying the exchange factor between permuted particles. In brief, this theory suggests that for q bosons, the commutation relations of the Heisenberg–Weyl algebra no longer hold. According to the commutation relations, there are two kinds of q bosons. One satisfies [41] (for simplicity we omit the particle index)

$$[b_q, b_q^\dagger]_q = b_q b_q^\dagger - q b_q^\dagger b_q = 1, \tag{1}$$

where $b_q^\dagger$ and b_q are creation and annihilation operators of q bosons, respectively. The deformation in Eq. (1) is non-symmetric under the transformation $q \to q^{-1}$, and so this kind of q boson is named as a non-symmetric q boson. The other satisfies

$$[c_q, c_q^\dagger]_q = c_q c_q^\dagger - q c_q^\dagger c_q = q^{-N'}, \tag{2}$$

where $c_q^\dagger$ and c_q are, respectively, creation and annihilation operators of q bosons, and N' is the number operator. It is easy to find that the deformation in Eq. (2) is symmetric under the transformation $q \to q^{-1}$, and so we call this kind of q boson a symmetric q boson. This deformation was first introduced [42] to give a realization of the "quantum groups" [43]. From then on, many papers were devoted to the investigation on this two kinds of q bosons even up to now [44–56,33,34,38], and these researches have greatly accelerated the development of the theory of q bosons. Although the commutation relations of the two kinds of q bosons are distinctly different, further investigation reveals that there is a close connection between