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Quantum discord near Anderson metal–insulator transition with correlated disorder

W.W. Cheng [a,b,c,*], J.X. Li [b], C.J. Shan [b], L.Y. Gong [a], S.M. Zhao [a]

[a] Institute of Signal Processing & Transmission, Nanjing University of Posts and Telecommunication, Nanjing 210003, China

[b] Department of Physics, Hubei Normal University, Huangshi 435002, China

[c] National Laboratory of Solid State Microstructures, Nanjing University, Nanjing 210093, China

HIGHLIGHTS

- Quantum discord exhibits a sudden transition around the boundary between the localized state and extended ones.
- Quantum discord does not depend on the system size L in the region of extended states.
- The results indicate that there exists a smooth crossover in the region $1 \lesssim \alpha \lesssim 2$ except for the critical point 2.

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ABSTRACT

Localization transitions in the one-dimensional Anderson model with long range correlated disorder are studied by means of quantum discord. The results show that the quantum discord exhibits a sudden transition around the boundary between the localized state and extended ones, and does not depend on the system size L in the region of extended states. This feature can depict an identical phase diagraph as well as the traditional method. As an another example, we also consider this quantum correlation in the random dimer model. It is then shown that the quantum discord can be a good quantity to detect the localization transition in these one-dimensional systems with correlated disorder.

1. Introduction

As one of the fundamental quantum phenomena in the nature, Anderson localization has attracted much attention in many branches of physics both in theoretical aspects and experimental ones [1–21]. For the one-dimensional disordered systems, it is well known that all eigenstates are localized and there are no mobility edges (MEs) separating the localized and extended states. However, many works have shown that there also exist extended states in some one-dimensional aperiodic systems [5,6], quasi-periodic systems [8,9] and disordered ones with correlated potentials [10–19,22,23]. Recently, this novel quantum phenomena has been realized experimentally [2–4], e.g., Billy et al. have reported the exponential localization of a Bose–Einstein condensate released into a one-dimensional waveguide in the presence of a controlled disorder created by laser speckle [2]. Traditionally, localization properties can be studied by judging Lyapunov coefficient, dynamics of wave function, multi-fractal properties, participation ratio, level statistics, and so on.

On the other hand, the concept of correlation, i.e., information of one subsystem to another in a composite system, is an underline element in the many-body theory of the condensed matter physics. For a quantum system, the correlations are

* Corresponding author at: Institute of Signal Processing & Transmission, Nanjing University of Posts and Telecommunication, Nanjing 210003, China. Tel.: +86 2583492417.

E-mail address: wwcheng@njupt.edu.cn (W.W. Cheng).

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often involved with the classical and quantum sources. Generally, the existence of quantum correlation can be depicted by entanglement which was pointed out by Schrödinger as the fundamental property of quantum theory. Since then, quantum entanglement has been regarded as the only kind of quantum correlation. In the past decades, this quantum correlation delivered a new perspective to investigate the quantum critical phenomena in the condensed physics [24–27]. However, many experimental works also indicate that there exist other quantum correlations which cannot be captured by entanglement. Recently, a so-called concept of quantum discord has been proposed [28], and has inspired much interest to study this quantum correlation as a probe to detect the quantum critical phenomena in varying spin models [29–34] and electron systems [35]. Specially, this quantum correlation has also been extended to detect the localization transition in the one-dimensional aperiodic systems and quasi-periodic systems [36,37], which determined on-site energies and MEs. In this work, from the perspective of quantum discord, we focus our attention on the localization transitions in two typical one-dimensional Anderson models with non-determined on-site energies (e.g., long range correlated disorder model and random dimer ones), in which the property of localization is still under debate. The results show that the quantum discord is a good quantity to detect the localization transition in these typical correlated disorder systems, which may shed some light on the understanding of localization transition for the other more complex systems from the view of quantum discord.

This work is organized as follows. In Section 2, we give a brief introduction to the concept of quantum discord $\mathcal{Q}_D$ and the Hamiltonian. In Section 3, the quantum discord and localization transition in the Anderson model with long range correlated disorder and random dimer ones are investigated carefully. Finally, we give a summary of our main results.

2. Model and formula

We consider a single particle moving in the one-dimensional lattice with L sites. The tight-binding Hamiltonian can be described by

$$H = \sum_i^L \epsilon_i c_i^\dagger c_i - t\sum_i^L (c_i^\dagger c_{i+1} + H.c.), \tag{1}$$

where ϵ_i is the on-site potential, t is the nearest-neighbor hopping integral, and can be set $t = 1$ without loss of generality. $c_i^\dagger$ (c_i) is a creation (annihilation) operator of the ith site. The Hamiltonian can be written as tridiagonal matrix on the site occupation basis $|n_1, n_2, \ldots, n_i, \ldots, n_L\rangle = c_1^{\dagger n_1} c_2^{\dagger n_2} \cdots c_i^{\dagger n_i} \cdots c_L^{\dagger n_L}|0\rangle$, where $n_i = 0, 1$, and $|0\rangle$ is the vacuum. For a single particle system, $\sum_{i=1}^L n_i = 1$. If we write $|i\rangle = |0, \ldots, 1_i, \ldots, 0\rangle = c_i^\dagger|0\rangle$, the general eigenstate $|\phi^\beta\rangle$ with eigenenergy E_β for the Hamiltonian is the superposition

$$|\phi^\beta\rangle = \sum_i^L \phi_i^\beta |i\rangle = \sum_i^L \phi_i^\beta c_i^\dagger |0\rangle, \tag{2}$$

where ϕ_i^β is the amplitude of the βth wave function at ith site. In the following, we diagonalize the tridiagonal matrix numerically with the open boundary condition to find all eigenenergies E_β and the corresponding eigenstates $|\phi^\beta\rangle$.

In the quantum physics, the total correlations between two subsystems are expressed by quantum mutual information,

$$\mathcal{L}(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}). \tag{3}$$

Here, $S(\rho) = \mathrm{Tr}(\rho \log_2 \rho)$ is the von Neumann entropy. And the classical correlation for such system can be defined as,

$$\mathcal{C}(\rho_{AB}) = S(\rho_A) - \min_{\{B_k\}} \sum_k p_k S(\rho_k), \tag{4}$$

where $\rho_k = (I_A \otimes B_k)\rho_{AB}(I_A \otimes B_k)$ is the post-measurement state of A after obtaining the outcome k on B with the probability $p_k = \mathrm{Tr}[(I_A \otimes B_k)\rho_{AB}(I_A \otimes B_k)]$. Then the pure quantum correlation described by the quantum discord can be defined as,

$$\mathcal{Q}(\rho_{AB}) = \mathcal{L}(\rho_{AB}) - \mathcal{C}(\rho_{AB}). \tag{5}$$

In order to obtain the quantum discord, we need to consider the two-site reduced density matrix $\rho_{i,j}$ obtained by tracing out all sites except sites i and j in the one-dimensional lattice. For the single particle system, there are two local states at each site, $|0\rangle$ and $|1\rangle$, corresponding to the state with (without) a particle at the ith site, respectively. On the basis of $|00\rangle_{ij}$, $|01\rangle_{ij}$, $|10\rangle_{ij}$ and $|11\rangle_{ij}$, the reduced density matrix for the sites i and j take the form,

$$\rho_{ij} = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & \rho_{44} \end{pmatrix}, \tag{6}$$