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Stochastic resonance in an extended FitzHugh–Nagumo system: The role of selective coupling

Claudio J. Tessone[a], Horacio S. Wio[b,*]

[a]Institut Mediterrani d'Estudis Avançats (IMEDEA-CSIC), Universitat de les Illes Balears, E07122 Palma de Mallorca, Spain
[b]Instituto de Física de Cantabria, Universidad de Cantabria and CSIC, E39005 Santander, Spain

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Abstract

Here we present a study of *stochastic resonance* (SR) in an extended FitzHugh–Nagumo system with a field dependent activator diffusion. We show that the system response (here measured through the output signal-to-noise ratio (SNR)) is enhanced due to the particular form of the non-homogeneous coupling. Such a result supports previous ones obtained in a simpler scalar reaction-diffusion system and shows that such an enhancement, induced by the field dependent diffusion -or selective coupling-, is a robust phenomenon.

Keywords: Stochastic resonance; Spatially-extended systems; Field dependent diffusion

1. Introduction

Stochastic resonance (SR) is one of the most interesting *noise-induced phenomena* that arises from the interplay between *deterministic* and *random* dynamics in a *nonlinear* system [1]. A large number of examples showing SR occur in *extended* systems: for example, diverse experiments were carried out to explore the role of SR in sensory and other biological functions [2] or in chemical systems [3]. These, together with the possible technological applications, motivated many recent studies showing the possibility of achieving an enhancement of the system response by means of the coupling of several units in what conforms an *extended medium* [4–6].

In previous works [5,6] we have studied the stochastic resonant phenomenon in extended systems, when transitions between two different spatial patterns occurs, exploiting the concept of the *non-equilibrium potential* (NEP) [7,8]: a Lyapunov functional of the associated deterministic system that, for non-equilibrium systems, plays a role similar to that of a thermodynamic potential in equilibrium thermodynamics. Such NEP characterizes the global properties of the dynamics: attractors, relative (or nonlinear) stability of these attractors, height of the barriers separating attraction basins and, in addition, allowing us to evaluate the

*Corresponding author. Fax: +34 942 200 935.
E-mail addresses: tessonec@imedea.uib.es (C.J. Tessone), wio@ifca.unican.es (H.S. Wio).

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transition rates among the different attractors. In another work [9] we have also shown that, for a scalar reaction–diffusion system with a density-dependent diffusion and a known form of the NEP, the non-homogeneous spatial coupling changes the effective dynamics of the system and contributes to enhance the SR phenomenon.

Here we report on a study of SR in an extended system: an array of FitzHugh–Nagumo [11] units, with a density-dependent (diffusive-like) coupling. The NEP for this system was found within the excitable regime and for particular values of the coupling strength [6]. In the general case, however, the form of the NEP has not been found yet. Nevertheless, the idea of the existence of such a NEP is always *underlying* our study. Hence, we have resorted to an study based on numerical simulations, analyzing the influence of different parameters on the system response. The results show that the enhancement of the signal-to-noise ratio (SNR) found for a scalar system [9] is robust, and that the indicated non-homogeneous coupling could clearly contribute to enhance the SR phenomenon in more general situations.

2. Theoretical framework

2.1. The model

For the sake of concreteness, we consider a simplified version of the FitzHugh–Nagumo [6,8,11] model. This model has been useful for gaining qualitative insight into the excitable and oscillatory dynamics in neural and chemical systems [10]. It consist of two variables, in one hand u, a (fast) activator field that in the case of neural systems represents the voltage variable, while in chemical systems represents a concentration of a self-catalytic species. On the other hand v, the inhibitor field, associated with the concentration of potassium ions in the medium (within a neural context), that inhibits the generation of the u species (in a chemical reaction). Instead of considering the usual cubic like nonlinear form, we use a piece-wise linear version

$$\varepsilon \frac{\partial u(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(D_u(u)\frac{\partial u}{\partial x}\right) + f(u) - v + \xi(x,t), \quad (1)$$

$$\frac{\partial v(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(D_v(v)\frac{\partial u}{\partial x}\right) + \beta u - \alpha v, \quad (2)$$

where $f(u) = -u + \Theta(u - \phi_c)$, and $\xi(x,t)$ is a δ-correlated white Gaussian noise, that is $\langle \xi(x,t)\rangle = 0$ and $\langle \xi(x,t)\xi(x',t')\rangle = 2\gamma\delta(x-x')\delta(t-t')$. Here γ indicates the noise intensity and ϕ_c is the "discontinuity" point, at which the piece-wise linearized function $f(u)$ presents a jump. In what follows, the parameters α and β are fixed as $\alpha = 0.3$ and $\beta = 0.4$. Finally, ε is the parameter that indicates the time-scale ratio between activator and inhibitor variables, and is set as $\varepsilon = 0.03$. We consider Dirichlet boundary conditions at $x = \pm L$. Although the results are qualitatively the same as those that could appear considering the usual FitzHugh–Nagumo equations, this simplified version allows us to compare directly with the previous analytical results for this system [6].

As in Ref. [9], we assume that the diffusion coefficient $D_u(u)$ is not constant, but depends on the field u according to $D_u(u) = D_u[1 + h\Theta(u - \phi_c)]$. This form implies that the value of $D_u(u)$ depends "selectively" on whether the field u fulfills $u > \phi_c$ or $u < \phi_c$. D_u is the value of the diffusion constant without such "selective" term, and h indicates the size of the difference between the diffusion constants in both regions (clearly, if $h = 0$ then $D_u(u) = D_u$ constant). $D_v(v)$ is the diffusion for the inhibitor v, that here we assume to be homogeneously constant.

It is worthwhile noting that when the parameter h is large enough, under some circumstances the coupling term might become negative. This is what is known as "inhibitory coupling" [12]. This is a very interesting kind of coupling that has attracted much attention in the last years, both in neural and chemical context, that we will not discuss here.

This system is known to exhibit two stable stationary patterns. One of them is $u(x) = 0$, $v(x) = 0$, while the other is one with non-zero values and can be seen in Fig. 1. We will denote with $P_0^{u,v}(x)$ and $P_1^{u,v}(x)$, the patterns for u and v fields. Further, we consider that an external, periodic, signal enters into the system