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Fermi statistics applied to a weakly excited column of granular particles in a vibrating bed

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Abstract

A one-dimensional experiment in granular dynamics is carried out to test the thermodynamic theory of weakly excited granular systems [Hayakawa and Hong, Phys. Rev. Lett. 78 (1997) 2764] where granular particles are treated as spinless Fermions. The density profile is measured and then fit to a granular Fermi distribution function, from which the global temperature of the system, T, is determined. Then the center of mass, $\langle z(T)\rangle$, and its fluctuations, $\langle \Delta z(T)^2\rangle$, are measured and plotted as functions of T. The granular Fermi function fits the density profile fairly well, with the value of T being reasonably close to the predicted value. The scaling behavior of $\langle z(T)\rangle$ and $\langle \Delta z(T)^2\rangle$ is in excellent agreement with the theory.

Keywords: Fermi statistics; Granular density profile; Weakly excited granular systems

1. Introduction

This paper is an experimental test of the theory presented and tested in preceding papers [1,2]. The thermodynamic theory of Hayakawa and Hong (HH) presented in Ref. [2] was tested in Ref. [1] with extensive molecular dynamics simulations. The purpose of this paper is to test the theory of HH experimentally for a one-dimensional vibrating granular system. A one-dimensional system is peculiar in the sense that randomness associated with collisions is suppressed in contrast with what happens in higher dimensions. Nevertheless, the Fermi-like statistics arising from hard core repulsions still apply to this simple one-dimensional system. As in Ref. [1], we determine the configurational statistics of a one-dimensional vibrating granular system by properly taking the ensemble average of the steady state. Then we measure physical quantities of the system and compare them to those predicted by HH. First, we measure the density profile and determine the dimensionless granular Fermi temperature,

$$T = \frac{T_f}{mgD},$$

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where T_f is the granular Fermi temperature, by fitting the density profile to the granular Fermi function. Second, we compare the measured granular Fermi temperature T to those predicted by the theory in Refs. [1,2]. Third, we measure the center of mass $\langle z(T) \rangle$ and the fluctuations of the center of mass $\langle \Delta z(T)^2 \rangle$ for the vibrating bed and test the scaling predictions of Ref. [2]. We will first summarize the theory presented by HH.

2. Background of Fermi-like statistics and thermodynamic theory of granular materials

The system being studied here is a dense, dissipative, nonequilibrium, granular system, where the mean free path of the grains is of the order of a few particle diameters. Hence, each particle may be considered to be effectively confined in a cage as in the free volume theory of a dense liquid [3]. In such a case, an observation has been made in Ref. [4] that the basic granular state is not a gas, but a solid or crystal, and thus, the effective thermodynamic theory based on the free energy argument may be more appropriate than the kinetic theory in studying this state. In such a case, the configurational statistics of the steady state may be determined by the variational method as the most probable or minimum free energy state.

To be more specific, consider the excitation of disordered granular materials confined in a box with vibrations of the bottom plate. The vibrations will inject energy into the system which cause the ground state to become unstable, and a newly excited state will emerge with an expanded volume. The time averaged configurational statistics of this new excited state have undergone structural distortions. However, the degree of distortions from the ground state may be small for a weakly excited state, possibly justifying the use of an effective thermodynamic theory based on the variational principle. Such a thermodynamic approach may be further justified by the following two experiments conducted previously.

1. *Weakly or moderately excited regime*: Clement and Rajchenbach (CR) [4] have performed an experiment with the vibrational strength, Γ, of the order one for a two-dimensional vibrating bed, using inclined side walls to suppress convections. Here, $\Gamma = A\omega^2/g$ with A and ω, the amplitude and frequency of the vibrating plane, and g, the gravitational constant. CR have found that the ensemble-averaged density profile as a function of height from the bottom layer obeys a universal function that is independent of the phase of oscillations of the vibrating plate. Namely, it is independent of the kinetics imposed on the system. One conceptually important point here is that the reference point of the density profile is not the bottom plate, but the bottom layer, which of course is fluidized.

2. *Highly excited regime*: Warr and Hansen (WH) [5] have performed an experiment on highly agitated, vertically vibrating beds of $\Gamma \approx 30-50$ using steel balls with a small coefficient of restitution. They have found that the collective behaviors of this vibrated granular medium *in a stationary nonequilibrium state* exhibits strong similarities to those of an atomistic fluid in *thermal equilibrium* at the corresponding particle packing fraction, in particular, in the two-point correlation function [6,7].

The results of both experiments indicate that for both moderate or highly excited systems, one-to-one correspondence seems to exist between *configurational* statistics of the nonequilibrium stationary state and the equilibrium thermal state. In fact, this is not so surprising considering that upon vibration, the granular materials expand and increase the volume of the system. In turn, this increase corresponds to a rise in the potential energy after the configurational average is appropriately taken. Then the problem reduces to a packing problem, and the temperature-like variable, T, can be associated to the vibrating bed. The existence of distinctive configurational statistics in the density profile of CR (and also in WH in a special case) appears to be fairly convincing evidence that kinetic aspects of the excited granular materials may be separated out from the statistical configurations. These observations are the basis of the thermodynamic theory proposed in Ref. [2]. Note that the Fermi-like statistics are essentially the macroscopic manifestation of the classical excluded volume effect and the anisotropy which causes the ordering of potential energy by gravity. The top surface of the granules plays the role of a Fermi surface, and the thin boundary layers that appear near the top layer upon excitation play the role of excited electrons of the Fermi gas in metals.

3. Thermodynamic theory of weakly excited granular systems

1. *Granular Fermi temperature*: In Ref. [2], the vibrating system was viewed from two different points of view. One may view it as a mechanical system, in which case the expansion is due to excitation induced by