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Scheme for cloning an unknown single qutrit state with assistance

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Abstract

We propose a scheme for cloning an unknown single qutrit state with assistance. The scheme includes a qutrit-state teleportation at the cloner's site. During this process different states (i.e., the maximally entangled or non-maximally entangled two-qutrit states) as quantum channel are taken into account. After the teleportation, to help the cloner Alice to reestablish the unknown state, the state preparer Victor should perform a single-qutrit measurement and tell her the outcome. In this scheme, the success probability of cloning the original state is determined by the used quantum channel.

Keywords: Quantum cloning; Quantum teleportation; Unitary operation

Manipulation and extraction of information has advanced very much since quantum information was proposed. In the process of developing quantum information theory, Wootters and Zurek [1] and Dieks [2] raised an interesting and very important problem that whether it is possible to copy a quantum state perfectly in the early 1980s, and furthermore, they approved it is impossible to do so. However, sooner or later, with the development of quantum information theory which includes quantum computer [3,4], quantum cryptography [5,6], quantum teleportation [7,8], quantum secret sharing (QSS) [9–11] and so on, people know although exact cloning is impossible, they can approximately clone an unknown state in the way of combining two different quantum information procedures. Since then, much attention has been paid to quantum clone. Research work on quantum clone was soon widely started up, and has got great development. As a result, various cloning machines have been proposed in the literature, theoretical and experimental as well [12–25].

The imperfect cloning may be divided into two main kinds: deterministic cloning and probabilistic cloning. If a cloning machine performs merely unitary operations, it is called deterministic cloning, since unitary evolution is deterministic. On the other hand, if a cloning machine performs measurements as well as unitary operations, with a post selection of the measurement results, it is called probabilistic cloning. For this kind of the imperfect cloning, some schemes have been proposed recently [26–30]. For example, in the year 2000 Pati [28] has proposed a feasible scheme for cloning an unknown *single-qubit* state with the state preparer's

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assistance through combining quantum teleportation and projective measurement. Using the similar method, in 2003 Chen and Wu [29] presented a protocol to probabilistically clone an unknown *single-qubit* state and its orthogonal complement state with assistance. Very recently Zhan [30] has proposed another scheme which can produce copies of an unknown *two-qubit* entangled state via two entangled particle pairs as the quantum channel. In this paper, similar to their works, we will propose a clone scheme with assistance to consider an unknown *single-qutrit* state case.

Suppose Alice owns a qutrit which is originally prepared by Victor in the state $|W\rangle_1 = \frac{1}{\sqrt{3}}(|0\rangle_1 + e^{i\xi_1}|1\rangle_1 + e^{i\xi_2}|2\rangle_1)$. However, Alice does not know what state this qutrit is in. Alice wants to clone this state with Victor's assistance. To achieve this goal, Alice first performs a usual teleportation in her site (say, with Bob). Suppose a maximally entangled two-qutrit state is used as the quantum channel linking Alice and Bob. Without loss of generality, suppose it is $|\phi\rangle_{23} = \frac{1}{\sqrt{3}}(|00\rangle_{23} + |11\rangle_{23} + |22\rangle_{23})$, and the qutrit 2 belongs to Alice and the qutrit 3 to Bob. So the initial quantum state of the whole system reads

$$\begin{aligned}|\Psi\rangle_{123} &= |W\rangle_1 \otimes |\phi\rangle_{23} \\ &= \tfrac{1}{3}(|000\rangle_{123} + |011\rangle_{123} + |022\rangle_{123} + e^{i\xi_1}|100\rangle_{123} + e^{i\xi_1}|111\rangle_{123} \\ &\quad + e^{i\xi_1}|122\rangle_{123} + e^{i\xi_2}|200\rangle_{123} + e^{i\xi_2}|211\rangle_{123} + e^{i\xi_2}|222\rangle_{123}).\end{aligned} \tag{1}$$

Alice performs a generalized Bell-state projective measurement on her qutrits 1 and 2. The generalized Bell states throughout this paper are defined as

$$|\Psi_{nm}\rangle = \sum_{j=0}^{2} e^{2\pi ijn/3}|j\rangle \otimes |(j+m) \bmod 3\rangle/\sqrt{3}, \tag{2}$$

where $n, m = 0, 1, 2$. Alice's generalized Bell-state projective measurement collapses the system state into one of the following nine possible results,

$$|\Psi_{00}\rangle_{12}\langle\Psi_{00}|\Psi\rangle = \tfrac{1}{3\sqrt{3}}|\Psi_{00}\rangle_{12}(|0\rangle + e^{i\xi_1}|1\rangle + e^{i\xi_2}|2\rangle)_3, \tag{3}$$

$$|\Psi_{01}\rangle_{12}\langle\Psi_{01}|\Psi\rangle = \tfrac{1}{3\sqrt{3}}|\Psi_{01}\rangle_{12}(|1\rangle + e^{i\xi_1}|2\rangle + e^{i\xi_2}|0\rangle)_3, \tag{4}$$

$$|\Psi_{02}\rangle_{12}\langle\Psi_{02}|\Psi\rangle = \tfrac{1}{3\sqrt{3}}|\Psi_{02}\rangle_{12}(|2\rangle + e^{i\xi_1}|0\rangle + e^{i\xi_2}|1\rangle)_3, \tag{5}$$

$$|\Psi_{10}\rangle_{12}\langle\Psi_{10}|\Psi\rangle = \tfrac{1}{3\sqrt{3}}|\Psi_{10}\rangle_{12}(|0\rangle + e^{-2\pi i/3}e^{i\xi_1}|1\rangle + e^{-4\pi i/3}e^{i\xi_2}|2\rangle)_3, \tag{6}$$

$$|\Psi_{20}\rangle_{12}\langle\Psi_{20}|\Psi\rangle = \tfrac{1}{3\sqrt{3}}|\Psi_{20}\rangle_{12}(|0\rangle + e^{-4\pi i/3}e^{i\xi_1}|1\rangle + e^{-8\pi i/3}e^{i\xi_2}|2\rangle)_3, \tag{7}$$

$$|\Psi_{11}\rangle_{12}\langle\Psi_{11}|\Psi\rangle = \tfrac{1}{3\sqrt{3}}|\Psi_{11}\rangle_{12}(|1\rangle + e^{-2\pi i/3}e^{i\xi_1}|2\rangle + e^{-4\pi i/3}e^{i\xi_2}|0\rangle)_3, \tag{8}$$

$$|\Psi_{21}\rangle_{12}\langle\Psi_{21}|\Psi\rangle = \tfrac{1}{3\sqrt{3}}|\Psi_{21}\rangle_{12}(|1\rangle + e^{-4\pi i/3}e^{i\xi_1}|2\rangle + e^{-8\pi i/3}e^{i\xi_2}|0\rangle)_3, \tag{9}$$

$$|\Psi_{12}\rangle_{12}\langle\Psi_{12}|\Psi\rangle = \tfrac{1}{3\sqrt{3}}|\Psi_{12}\rangle_{12}(|2\rangle + e^{-2\pi i/3}e^{i\xi_1}|0\rangle + e^{-4\pi i/3}e^{i\xi_2}|1\rangle)_3, \tag{10}$$

$$|\Psi_{22}\rangle_{12}\langle\Psi_{22}|\Psi\rangle = \tfrac{1}{3\sqrt{3}}|\Psi_{22}\rangle_{12}(|2\rangle + e^{-4\pi i/3}e^{i\xi_1}|0\rangle + e^{-8\pi i/3}e^{i\xi_2}|1\rangle)_3. \tag{11}$$

Then Alice informs the state receiver Bob of her measurement results via a classical channel. Bob then reconstructs the state $|W\rangle$ at his place in terms of Alice's message. Alice sends the qutrit 1 back to Victor. Since Victor as the state preparer knows the parameters ξ_1 and ξ_2 of the original state $|W\rangle_1$, he performs