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On retrial queueing model with fuzzy parameters

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Abstract

This work constructs the membership functions of the system characteristics of a retrial queueing model with fuzzy customer arrival, retrial and service rates. The α-cut approach is used to transform a fuzzy retrial-queue into a family of conventional crisp retrial queues in this context. By means of the membership functions of the system characteristics, a set of parametric non-linear programs is developed to describe the family of crisp retrial queues. A numerical example is solved successfully to illustrate the validity of the proposed approach. Because the system characteristics are expressed and governed by the membership functions, more information is provided for use by management. By extending this model to the fuzzy environment, fuzzy retrial-queue is represented more accurately and analytic results are more useful for system designers and practitioners.
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1. Introduction

Retrial queueing models are characterized by the feature that the arriving customers encountering the server busy will have to leave the service area and repeat their request for service after a random period. During trials, the blocked customer joins a pool of unsatisfied customers called orbit, which the inter-retrial time is independent of the number of customers in the orbit. Such queueing systems play important roles in the analysis of many telephone-switching systems, telecommunication networks and computer systems. Review of retrial queue literature could be found in Refs. [1–4]. A number of applications of retrial queues in science and engineering can be found in Ref. [5]. Recently, Diamond and Alfa [6] constructed a method for approximating the stationary distribution and waiting time moments of an $M/\mathrm{PH}/1$[1] retrial queue with phase type inter-retrial times. The $\mathrm{BMAP}/G/1$ retrial system with search for customers immediately on termination of service was

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[1]Kendall's shorthand notation $a/b/c/d$ is widely used to describe queueing models. In this notation, a specifies the interarrival time (arrival process), b specifies the service time, c is the number of servers, and d is the restriction of system capacity. In many situations only the first three symbols are used. Current practice is to omit the service-capacity symbol if no restriction is imposed ($d = \infty$). The following symbols are used: M for the exponential distribution (for memoryless), G for a general distribution, D for deterministic, PH for PH distribution, …, etc. In addition, BMAP represents batch Markovian arrival process.

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studied by Dudin et al. [7], in which the inter-retrial time is followed by an exponential distribution and the duration of search is characterized by a generally distributed random variable. Lopez-Herrero [8] presented the explicit formulae for the probabilities of the number of customers (denoted by I) being served in a busy period, and an explicit expression for the second moment of I for the $M/G/1$ retrial queueing system is also been given. Lopez-Herrero further employed the principle of maximum entropy to estimate the distributions of I.

In the literature described above, the inter-arrival times, inter-retrial times and service times of customers are required to follow certain probability distributions with fixed parameters. However, in many real-world applications, the parameter distributions may only be characterized subjectively; that is, the arrival, retrial and service patterns are typically described in everyday language summaries of central tendency, such as the mean arrival rate is around $10\,\mathrm{min}^{-1}$, the mean retrial rate is about $5\,\mathrm{min}^{-1}$ or the mean service rate is approximately $8\,\mathrm{min}^{-1}$, rather than with complete probability distributions. In other words, these system parameters are both possibilistic and probabilistic. Thus, fuzzy retrial queues would be potentially much more useful and realistic than the commonly used crisp retrial queues (see also [9,10]). By extending the usual crisp retrial queues to fuzzy retrial queues in the context, these queuing models become appropriate for a wider range of applications.

Li and Lee [9] investigated the analytical results for two typical fuzzy queues (denoted $M/F/1/\infty$[2] and FM/FM/$1/\infty$, where F represents fuzzy time and FM represents fuzzified exponential distributions) using a general approach based on Zadeh's extension principle (see also [11,12]), the possibility concept and fuzzy Markov chains (see [13]). A useful modelling and inferential technique would be to apply their approach to general fuzzy queuing problems. However, their approach is complicated and not suitable for compu tational purposes; moreover, it cannot easily be used to derive analytical results for other complicated queuing systems (see also [14]). Negi and Lee [14] proposed a procedure using α-cuts and two-variable simulation to analyse fuzzy queues (see also [15]). Unfortunately, their approach provides only crisp solutions; i.e., it does not fully describe the membership functions of the system characteristics. Using parametric programming, Kao et al. [16] constructed the membership functions of the system characteristics for fuzzy queues and successfully applied them to four simple fuzzy queue models: $M/F/1/\infty$, $F/M/1/\infty$, $F/F/1/\infty$ and FM/FM/$1/\infty$. Recently, Chen [17,18] developed FM/FM/$1/L$ and FM/FM$^{[K]}/1/\infty$ fuzzy systems using the same approach.

All previous research on fuzzy queuing models is focused on ordinary queues with one or two fuzzy variables. In this paper, we develop an approach that provides system characteristics for the retrial queues with three fuzzy variables: fuzzified exponential arrival, retrial and service rates. Through α-cuts and Zadeh's extension principle, we transform the fuzzy retrial queues to a family of crisp retrial queues. As α varies, the family of crisp retrial queues is described and solved using parametric non-linear programming (NLP). The NLP solutions completely and successfully yield the membership functions of the system characteristics. Although an explicit closed-form expression for the membership function is very difficult to obtain in the case of three fuzzy variables, we develop a characterization that yields closed-form expressions when interval limits are invertible.

The remainder of this paper is organized as follows. Section 2 presents the system characteristics of standard and fuzzy retrial queuing models. In Section 3, a mathematical programming approach is developed to derive the membership functions of the system characteristics. To demonstrate the validity of the proposed approach, a realistic numerical example is described and solved. Discussion is provided in Section 4 and conclusions are drawn in Section 5. For notational convenience, our model in this paper is hereafter denoted as FM/FM/1/1-(FR), where FR represents the fuzzified exponential retrial rate, and the first 1 represents the single server and the second 1 represents the system capacity.

[2]Kendall's notation combines with fuzzy time. Some examples are as follows: $M/F/1/\infty$—exponential interarrival time and general fuzzy service time; FM/FM/$1/\infty$—fuzzified exponential interarrival time and fuzzified exponential service time. For FM/FM$^{[K]}/1/\infty$ system, the second symbol represents fuzzified exponential service time that the waiting units (for service in the queue) are served K at a time, except when less than K are in the queue and ready for service, at which time all units are served.

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