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Controlling chaos in an economic model

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Abstract

A Cournot duopoly, with a bounded inverse demand function and different constant marginal production costs, can be modeled as a discrete-time dynamical system, which exhibits complex bifurcating and chaotic behaviors. Based on some essential features of the model, we show how bifurcation and chaos can be controlled via the delayed feedback control method. We then propose and evaluate an adaptive parameter-tuning algorithm for control. In addition, we discuss possible economic implications of the chaos control strategies described in the paper.

Keywords: Chaos; Control; Delayed feedback; Cournot; Oligopoly

1. Introduction

Since the demonstration of Strotz et al. [1] that chaotic dynamics can exist in an economics model, tremendous efforts have been devoted to investigating this kind of complex behaviors in various economic systems. Significant and transparent theoretical insights have been gained. Recently, it has also been shown that even oligopolistic markets may become chaotic under certain conditions [2,3].

Oligopoly, with a few firms in the market, is an intermediate structure between the two opposite cases of monopoly and perfect competition. Even the duopoly situation in an oligopoly of two producers can be more complex than one might imagine since the duopolists have to take into account their actions and reactions when decisions are made. Oligopoly theory is one of the oldest branches of mathematical economics dated back to 1838 when its basic model was proposed by Cournot [3]. Research reported in Ref. [4] suggests that a Cournot adjustment process of output might be chaotic if the reaction functions are non-monotonic. This result was purely mathematical without substantial economic implication until Puu [5] provided one kind of economic circumstances, i.e., iso-elastic demand with different constant marginal costs for the competitor, under which meaningful unimodal reaction functions were developed. Since then, various modifications have been made by numerous economists. Rosser [6] has a good state-of-the-art review of the theoretical development of complex oligopoly dynamics.

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Unstable fluctuations have always been regarded as unfavorable phenomena in traditional economics. Because chaos means unpredictable events in the long time, it is considered to be harmful by decision-makers in the economy. Research on controlling chaos in economic models has already begun and several methods have been applied to the Cournot model, such as the OGY chaos control method [7,8], the pole placement method [9]. These approaches require exact system information before their implementation. That means, to make an accurate decision, the government or oligolists have to possess enormous amounts of relevant economic data, which is impractical or very costly. In contrast, the delayed feedback control (DFC) method [10] can be easily applied without requiring any system information although it has some other drawbacks [11].

In the present paper, the Cournot–Puu model is considered. After describing the nonlinear dynamics of the economic model, system state and parameter DFCs of chaos are investigated. Then, an adaptive approach to tuning the key system parameter, i.e., the marginal cost according to the observed output oscillation, is proposed to stabilize the output quantity produced by a firm at a desired value. Finally, possible economic implications corresponding to the proposed control strategies will be discussed.

The paper is organized as follows. In Section 2, the nonlinear economical model and its dynamic properties are described. Section 3 is devoted to controlling chaos by using the delayed feedback and the proposed adaptive methods. Theoretical analysis and numerical simulations are included. Conclusions are finally presented in Section 4.

2. Cournot–Puu model

Alongside several interesting variants of the oligopoly theory emerged from criticism, the Cournot model was gradually evolved to be the core. In this paper, we particularly discuss the case of duopoly, that is, two suppliers facing many consumers on the demand side. Denote the two firms by F_1 and F_2, which produce quantities q_1 and q_2, respectively. To determine its production in time period $t+1$, firm F_1 has to do two things. First, it has an expectation on the quantity of the other firm F_2 in time period $t+1$. This expectation might, for example, depend on its own quantity and the quantity of the other firm, both produced in the previous time period, that is,

$$q_2^e(t+1) = E_2(q_1(t), q_2(t)), \tag{1}$$

where e denotes an expected value. Second, it has to determine the production that can maximize its own profit under the above expectation. Let $U_1(q_1, q_2^e)$ be the profit of firm F_1. From $\partial U_1/\partial q_1 = 0$, one obtains the quantity that leads to the maximal profit in time period $t+1$, as

$$q_1(t+1) = V_{1,maxU_1}(q_2^e(t+1)).$$

As seen from (1), $q_1(t+1)$ generally depends on both $q_1(t)$ and $q_2(t)$, that is,

$$q_1(t+1) = f_1(q_1(t), q_2(t)).$$

The same is true for firm F_2, i.e.,

$$q_2(t+1) = f_2(q_1(t), q_2(t)).$$

Here, $f_i(\cdot)$ $(i = 1, 2)$ are called reaction functions.

Next, we derive the specific expressions of the above reaction functions under the assumptions of Cournot and Puu [3].

Cournot assumption. Each firm expects its rival to offer the same quantity for sale in the current period as it did in the preceding period.

According to this assumption, Eq. (1) becomes $q_2^e(t+1) = q_2(t)$. Consequently, the general reaction functions are changed to

$$q_1(t+1) = f_1(q_2(t)), \quad q_2(t+1) = f_2(q_1(t)). \tag{2}$$

Although Cournot's assumption may seem rather simplistic, it did produce a more interesting consequence than its sophisticated revisions. Rand [4] shows that the Cournot model with non-monotonic reaction