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Nonlinear-map model for split effect on vehicular traffic through periodic signals

Takashi Nagatani*

Department of Mechanical Engineering, Division of Thermal Science, Shizuoka University, Hamamatsu 432-8561, Japan

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Abstract

We study the effects of both split and cycle time on dynamical behavior of vehicles moving through a sequence of traffic lights on a highway, where the traffic lights turn on and off periodically. The dynamical model of vehicular traffic controlled by signals is expressed in terms of a nonlinear map. The vehicle exhibits complex behavior with varying split and cycle time. The tour time between signals shows a self-similar behavior. When split s_p is lower than 0.5, vehicular traffic shows a similar behavior as that of $s_p = 0.5$, while vehicular traffic of $s_p > 0.5$ is definitely different from that of $s_p \leqslant 0.5$. The algebraic expression among the tour time, cycle time, and split is derived.

Keywords: Traffic dynamics; Signal control; Nonlinear map; Self-similarity; Complex system

1. Introduction

Recently, transportation problems have attracted much attention in the fields of physics [1–5]. The traffic flow, pedestrian flow, and bus-route problem have been studied from a point of view of statistical mechanics and nonlinear dynamics [6–23]. Interesting dynamical behaviors have been found in the transportation system. Jams and chaos are typical signatures of the complex behavior of transportation [24,25].

Mobility is nowadays one of the most significant ingredients of a modern society. In urban traffic, vehicles are controlled by traffic lights to give priority for a road because city traffic networks often exceed the capacity. Brockfeld et al.[26] have studied optimizing traffic lights for city traffic by using a CA traffic model. They have clarified the effect of signal control strategy on vehicular traffic. Also, they have shown that the city traffic controlled by traffic lights can be reduced to a simpler problem of a single-lane highway. Sasaki and Nagatani [27] have investigated the traffic flow controlled by traffic lights on a single-lane roadway by using the optimal velocity model. They have derived the relationship between the road capacity and jamming transition.

Traffic depends highly on both cycle time and split where the cycle time is the period of a traffic light and the split of signal is the fraction of green time to the signal period. Until now, one has studied the periodic traffic

*Fax: +81 53 478 1048.
E-mail address: tmtnaga@ipc.shizuoka.ac.jp.

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controlled by a few traffic lights. It has been concluded that the periodic traffic does not depend on the number of traffic lights [26,27]. Very recently, a few works have been done for the traffic of vehicles moving through an infinite series of traffic lights. The effect of cycle time on vehicular traffic has been classified [28–30]. However, the split effect has not been done for the vehicular traffic through a series of signals.

In this paper, we study the dynamical behavior of a single vehicle moving through an infinite series of traffic lights. The signals are periodically positioned with a constant distance on a single-lane roadway, controlled by a synchronized strategy, and turn on or off with a cycle time and a split. We present an extended model of nonlinear map to take into account both cycle time and split of traffic signals. We investigate the dynamical behavior of a single vehicle by iterating the nonlinear map. We clarify the dynamical states of a single vehicle through a sequence of traffic lights by varying both cycle and split time . We show that an algebraic relationship holds among tour time, cycle time, and split.

2. Nonlinear-map model

We consider the flow of vehicles going through an infinite series of traffic lights. Each vehicle passes freely over other vehicles. Then, each vehicle does not depend on the other and is uncorrelated with the other vehicles. Therefore, we consider the dynamical behavior of a single vehicle. The traffic lights are periodically positioned with distance l. The vehicle moves with a mean speed v between a traffic light and its next light. The traffic lights are numbered, from upstream to downstream, as 1, 2, 3,…, n, $n+1$,… . In the synchronized strategy, all the traffic lights change simultaneously from red (green) to green (red) with a fixed time period $(1-s_p)t_s(s_pt_s)$. The period of green is s_pt_s and the period of red is $(1-s_p)t_s$. Time t_s is called the cycle time and fraction s_p represents the split which indicates the ratio of green time to cycle time. When a vehicle arrives at a traffic light and if the traffic light is red, the vehicle stops at the position of the traffic light. Then, when the traffic light changes from red to green, the vehicle goes ahead. On the other hand, when a vehicle arrives at a traffic light and if the traffic light is green, the vehicle does not stop and goes ahead without changing speed.

We define the arrival time of the vehicle at traffic light n as $t(n)$. The arrival time at traffic light $n+1$ is given by

$$t(n+1) = t(n) + l/v + (r(n) - t(n))H(t(n) - (\mathrm{int}(t(n)/t_s)t_s) - s_pt_s),$$

with

$$r(n) = (\mathrm{int}(t(n)/t_s) + 1)t_s, \tag{1}$$

where $H(t)$ is the Heaviside function: $H(t)=1$ for $t\geqslant 0$ and $H(t)=0$ for $t<0$. $H(t)=1$ if the traffic light is red, while $H(t)=0$ if the traffic light is green. l/v is the time it takes for the vehicle to move between lights n and $n+1$. $r(n)$ is such time that the traffic light just changed from red to green. The third term on the right hand side of Eq. (1) represents such time that the vehicle stops if traffic light n is red. The number n of iteration increases one by one as the vehicle moves through the traffic light. The iteration corresponds to the going ahead on the highway. Eq. (1) is the extended version of nonlinear map model for split $s_p=0.5$.

By dividing time by the characteristic time l/v, one obtains the nonlinear equation of dimensionless arrival time:

$$T(n+1) = T(n) + 1 + (R(n) - T(n))H(T(n) - (\mathrm{int}(T(n)/T_s)T_s) - s_pT_s)$$

with

$$R(n) = (\mathrm{int}(T(n)/T_s) + 1)T_s, \tag{2}$$

where $T(n) = t(n)v/l$, $R(n) = r(n)v/l$, and $T_s = t_sv/l$. Thus, the dynamics of the vehicle is described by the nonlinear map (2). The motion of a vehicle depends on both dimensionless cycle time $T_s = t_sv/l$ and split s_p.

Generally, the length between two traffic lights or the mean speed of the vehicle vary from traffic light to traffic light. It is necessary and important to take into account the fluctuation of the length or speed. One can take into account the fluctuation as a noise. Here, we restrict ourselves to the vehicular traffic moving with constant speed through periodic signals.