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Models for random graphs with variable strength edges

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Abstract

Two models for random graph formation are introduced that use a set of vertices with an associated set of vectors. A random process determines whether edges will be formed or clusters of connected vertices destroyed: edge-formation between vertices with similar vectors is preferred, and cluster destruction is controlled by picking an edge at random, with the probability of destruction being greater if the edge connects two vertices with dissimilar vectors. Differential equations for edge-strength and cluster-size distributions are derived and presented, and solutions to these equations are compared with numerical simulations of the models. The models are shown to have robust power-law cluster-size distributions for all parametric variations of the model, with an exponent of $-\frac{5}{2}$. The edge-strengths are shown to have an approximately Gaussian distribution, which does not vary with cluster size.

Keywords: Herding; Decision processes; Complex networks

1. Introduction

The aim of the research described in this paper is to investigate the advantages and disadvantages of different structures for communication within large organizations; specifically, it is concerned with modelling spontaneously formed, co-operative, autonomous groups, known as Agile Mission Groups, in a military setting, and whether non-hierarchical organized structures can be more efficient at accomplishing certain tasks than traditional hierarchical ones.

It takes as its starting point [1–3]. A model for co-operation between traders in a financial market was set out in Ref. [1] and this model was solved exactly in Refs. [2,3]. In essence, the model allows traders to associate with one another randomly, sharing information; at some point the information is used by the traders, at which point the association between them, provided by the shared information, dissolves. This model was found to obey a modified power law.

Added to this model was the concept of tag-based co-operation set out in Refs. [4–8]. Tag-based co-operation was conceived in Refs. [5–8] as a sociological model to explain spontaneous co-operation between individuals with no known kinship relations and no expectation of return benefits by recognition of superficial shared "tags" or markers. This was extended to a practical application in peer-to-peer filesharing networks in

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Ref. [4]. The tag-based approach was adopted because it provides a simple, practical means of allowing autonomous agents with similar purposes to co-operate. Agents in a military scenario must share information in order to work effectively, but excessively broad sharing can be damaging. Information is best shared by agents with a similar purpose.

In this paper, we model a similar clustering strategy to that employed in Refs. [1–3], but encouraging links between agents with similar purposes and discouraging links between agents with different purposes. It was then decided to use the same kind of information sharing and use as in Refs. [1–3]. The first of these two is explored in this paper.

This question is investigated by creating a graph where each vertex corresponds to an agent. Each vertex has an associated vector, with a fixed number of entries filled. Agents with similar purposes are modelled by vertices with similar vectors. Edges are created between vertices with a probability proportional to the inner product of their vectors; the greater this inner product, the stronger the edge formed.

An edge can also be picked at random, and the connected cluster of vertices (that is, the set of all vertices connected to this edge by an unbroken path of edges) is destroyed with a probability of the edge strength subtracted from unity. Cluster destruction and edge-formation occur as the result of a random process, whose proportions are governed by a parameter.

Random graphs with power-law cluster distributions over several orders of magnitude have been observed in real-world situations such as the World Wide Web, the Internet, and within the set of actors contained in the Internet Movie Database [9–18].

This paper deals only with the random formation of groups, or clusters. Since it models individuals as vertices in a random undirected graph, an edge's existence implies the possibility of communication between two individuals; similar vectors are taken to imply a commonality of purpose between individuals; thus, it is desirable that vertices with similar associated vectors should be connected by an edge. Actual communication will be investigated in future work.

We examine the efficiency with which groups form, and the sizes of groups that do form. We also examine the distribution of edges of different strengths within the model.

2. Methods

Two models are introduced in this paper. In both models, we start with a set U of N vertices. Each vertex $u_i \in U$ contains a data vector x_i with m entries, s of which are $1/\sqrt{s}$, and the remainder of which are zero. Consequently, there are $\binom{m}{s}$ possible data vectors. Each vertex has a randomly chosen vector, and each vector may be selected with equal probability.

Vertices are then connected by a random process, and clusters created thereby are destroyed by a different random process. At each timestep, the model chooses whether to create edges or destroy clusters; the relative proportions are determined by a parameter p, which may be varied. With probability p the edge-formation process occurs; with probability $1-p$ the cluster-destruction process takes place.

The random process for edge-formation is the same in both models under consideration. Two vertices, u_i and u_j, are selected randomly, with uniform probability and without replacement, from the set of vertices. The inner product $x_i.x_j$ of the data vectors is taken. This is then used as the probability that an edge will be created between the vertices. From the definition of x_i given, it may be seen that this probability is r/s where r is some integer between 0 and s. This fact was exploited for ease of computation.

Within this model, both loops (where an edge has the same vertex at both ends) and multigraphs (where many edges may connect the same pair of vertices) are allowed.

The set E of edges thus formed varies in size as the model evolves. Each edge $e_i \in E$ has a *strength*, f_i which is the inner product of the data vectors corresponding to the two vertices joined by the edge.

The random process for cluster-destruction is determined by this strength. The two models differ in their approach.

Model 0 selects an edge e_i at random, with uniform probability, from the set E. With probability $1-f_i$ the cluster containing the edge is destroyed.