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Creation of excitations from a uniform impurity motion in the condensate

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HIGHLIGHTS

- We show creation of excitations from a uniform impurity motion in the condensate.
- We analytically derive different scaling behaviors for the transient excitations.
- We propose a new way of observing critical behaviors in the condensate.

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ABSTRACT

We investigate a phenomenon of creation of excitations in the homogeneous Bose–Einstein condensate due to an impurity moving with a constant velocity. A simple model is considered to take into account dynamical effects due to motions of the impurity. Based on this model, we show that there can be a finite amount of excitations created even if velocity of the impurity is below Landau's critical velocity. We also show that the total number of excitations scales differently for large time across the speed of sound. Thus, our result dictates the critical behavior across Landau's one and validates Landau's intuition to the problem. We discuss how Landau's critical velocity emerges and its validity within our model.

1. Introduction

Superfluidity has been one of fascinating phenomena in condensed matter physics, and has still remained as the central subject of research since its discovery. There exist various definitions and criteria whether given a system exhibits superfluidity or not, see for example Ref. [1].

It was Landau who gave a major breakthrough to understand this phenomenon based on the phenomenological theory, known as the two-fluid model. In his seminal paper in 1941, he also gave a simple yet powerful argument to determine a critical speed below which a fluid can move without any dissipation [2,3]. This critical speed, called Landau's critical velocity, is defined by

$$v_{\text{critical}} = \min_{\vec{p}} \left(\frac{\epsilon(\vec{p})}{|\vec{p}|} \right), \tag{1}$$

where $\epsilon(\vec{p})$ is an excitation spectrum of the fluid, and $\vec{p}$ is the momentum of excitations in the fluid. When this critical velocity is non-zero, the fluid supports dissipation-less motion and hence, the phenomenon of superfluidity occurs. As an example consider a fluid whose excitation spectrum is linear in the momentum $p = |\vec{p}|$, i.e., $\epsilon(\vec{p}) = c_s p$, the above critical velocity is equal to the speed of sound of the fluid c_s.

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The above critical velocity was originally obtained for relative motion of a fluid with respect to a container. Similar argument holds when considering a motion of an impurity (an obstacle) in the fluid and one can show that if the impurity moves slower than Landau's critical velocity, the steady-state excitation rate is zero, see for example, Ref. [4]. This result is often phrased as "the fluid cannot be excited if the impurity speed is below Landau's critical speed". This implies that an impurity in a fluid can move without friction if its velocity is below Landau's critical velocity, thus phenomenon of superfluidity. In fact, two kinds of forces to show the superfluidity need to be distinguished: One is a drag force onto the fluid when it moves against the impurity, and the other is a force acting on the impurity from the fluid. These two forces may or may not be same depending on details of a given model, i.e., a motion of the impurity, a coupling between the fluid and the impurity, and so on.

Other aspect of Landau's critical velocity is to infer the stability of a fluid as Landau discussed in his paper originally. Above this critical velocity, there can be dissipation resulting in a unstable configuration for the fluid. This instability is usually referred to as Landau's instability. There exists more refined instability criterion from analysis of the solution to non-linear partial different equations. Recently, Kato and Watabe proposed a unified criterion for the stability of a fluid based on the scaling behaviors of the autocorrelation function of the local density [5–7]. As will be discussed later, some of our results agree with theirs, whereas we are asking a different question and are examining a different physical quantity.

It is important to remind ourselves the following fact regarding Landau's argument: That is his derivation is purely *kinematical* and *classical*. There is no guarantee that we can apply it to understand dynamical and quantum aspects of this phenomenon. Another remark is that the Galilei invariance is a crucial assumption for the derivation and one cannot a priori apply this criterion for inhomogeneous systems.

Bose–Einstein condensates (BECs) are expected to show a phenomenon of superfluidity and there have been experimental efforts to examine superfluidity of BECs [8–12]. However, the results were rather surprising showing the critical speed was much below Landau's one. There were many theoretical analyses on this issue in past to understand this discrepancy [13–18]. Due to the experimental limitation, indeed some of assumptions in Landau's derivation are not satisfied and thus one needs to analyze many-body problem directly to calculate real critical velocity if it exists. Drag forces onto BECs when an impurity is immersed were also studied in Refs. [19–23]. In Ref. [19], it was concluded that there could be a finite drag force onto BECs at arbitrary small velocity [19–23]. If this statement is true, Landau's criterion is incorrect and BECs cannot show superfluidity in the traditional sense explained in textbooks.

It is our main motivation here to examine in which sense and under what conditions Landau's criterion becomes meaningful and one can use it to check the onset of superfluidity. Through this analysis we also wish to resolve some of disagreements in the previous studies. For this purpose, we ask a simple, yet a fundamental question: How many Bogoliubov excitations (bogolons) are created from a single impurity moving with a given constant speed v in BECs. In this paper, we analyze a point-like impurity immersed in the homogeneous BEC with spatial dimension three. To concentrate on the effect of impurity only, we consider the system of the condensate with the impurity at zero temperature in the thermodynamic limit. Since this is the situation where impurity version of Landau's criterion seems to hold, one expects: There cannot be any excitations created in BEC for $v < c_s$ where c_s is the speed of sound of the BEC. As will be analyzed in this paper, this naive intuition is incorrect and finite amount of bogolons are created at all velocity below the speed of sound. However, we also show that its scaling behavior in time completely changes across Landau's critical velocity. In this way, we give a different interpretation of Landau's critical velocity for the BEC, which was not addressed before.

This paper is organized as follows. Section 2 summarizes the model of impurity motion in the BEC and its solution. We then analyze asymptotic behaviors of excitations created in the BEC in Section 3 and comparisons with numerical analyses are also discussed. Section 4 discusses and compares our results with previously known results. We close the paper with brief summary and outlook in Section 5.

2. Impurity model and creation of Bogoliubov excitations

We consider a weakly interacting homogeneous condensate at zero temperature. The system is described by the Bogoliubov Hamiltonian [24]:

$$H_{\rm B} = \int d^3x\, \hat{\psi}^\dagger(\vec{x}, t)\left(-\frac{\hbar^2\,\vec{\nabla}^2}{2M}\right)\hat{\psi}(\vec{x}, t) + \frac{g}{2}\int d^3x\, \hat{\psi}^\dagger(\vec{x}, t)\hat{\psi}^\dagger(\vec{x}, t)\hat{\psi}(\vec{x}, t)\hat{\psi}(\vec{x}, t), \tag{2}$$

where M is the mass of bosons, g is the coupling constant between bosons, and $\hat{\psi}(\vec{x}, t)$ is the free boson field operator satisfying the equal-time canonical commutation relationship. To describe effects of an impurity in BEC, we analyze the following interaction Hamiltonian,

$$H_{\rm int} = g_{\rm i}\int d^3x\, \rho_{\rm i}(\vec{x} - \vec{\zeta}(t))\hat{\psi}^\dagger(\vec{x}, t)\hat{\psi}(\vec{x}, t). \tag{3}$$

Here $g_{\rm i}$ is the coupling constant between bosons and the impurity, $\rho_{\rm i}(\vec{x})$ is the density of the impurity at position $\vec{x}$, and $\vec{\zeta}(t)$ is a given trajectory for the impurity. This impurity model cooperates the effect of local density–density interaction between the massive bosons and the impurity. For a point-like impurity, we use an approximation $\rho_{\rm i}(\vec{x}) = \delta(\vec{x})$ to simplify the result. The effect of impurity size will be discussed in Section 4.