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Three-phase theory of city traffic: Moving synchronized flow patterns in under-saturated city traffic at signals

Boris S. Kerner*

Physik von Transport und Verkehr, Universität Duisburg–Essen, 47048 Duisburg, Germany

HIGHLIGHTS

- Three-phase traffic theory of city traffic.
- Moving synchronized flow patterns (MSP) at traffic signal.
- Random features of MSP emergence and evolution.
- Effect of MSP characteristics on green-wave breakdown.

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ABSTRACT

Three-phase traffic flow theory of city traffic has been developed. Based on simulations of a stochastic microscopic traffic flow model, features of moving synchronized flow patterns (MSP) have been studied, which are responsible for a random time-delayed breakdown of a green-wave (GW) organized in a city. A possibility of GW control leading to the prevention of GW breakdown has been demonstrated. A diagram of traffic breakdown in under-saturated traffic (transition from under- to over-saturated city traffic) at the signal has been found; the diagram presents regions of the average arrival flow rate, within which traffic breakdown can occur, in dependence of parameters of the time-function of the arrival flow rate or/and signal parameters. Physical reasons for a crucial difference between results of classical theory of city traffic and three-phase theory are explained. In particular, we have found that under-saturated traffic at the signal can exist during a long time interval, when the average arrival flow rate is larger than the capacity of the classical theory; the classical capacity is equal to a minimum capacity in three-phase theory. Within a range of the average arrival flow rate between the minimum and maximum signal capacities, under-saturated traffic is in a metastable state with respect to traffic breakdown. We have distinguished the following possible causes for the metastability of under-saturated traffic: (i) The arrival flow rate during the green phase is larger than the saturation flow rate. (ii) The length of the upstream front of a queue at the signal is a finite value. (iii) The outflow rate from a MSP (the rate of MSP discharge) is larger than the saturation flow rate.

1. Introduction

In highway traffic, complex spatiotemporal self-organization phenomena of traffic breakdown and resulting traffic congestion are observed [1–3]. However, as explained in a recent review [4], generally accepted classical traffic flow theories and models reviewed in Ref. [1] cannot explain the set of the fundamental empirical features of traffic breakdown at a highway bottleneck. This questions the application of the generally accepted classical traffic flow theories for the optimization, control, and management of *highway* traffic networks [4].

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* Tel.: +49 711 4793774.
E-mail address: boris.kerner@uni-due.de.

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In contrast with highway traffic, in the classical theories of city traffic it is assumed that no self-organization phenomena can occur in city traffic (see Refs. [5–20]).[1] This is because vehicles must stop during the red signal phase, i.e., traffic signals constitute massive deterministic perturbations in city traffic. The classical theories of city traffic [5–20] are the basis for a variety of signal control systems, in particular, for an arterial progressive control during which vehicles should travel unimpeded in a city called as *green wave* (GW) [19].

Free flow at a bottleneck due to traffic signal in a city intersection is associated with *under-saturated city traffic*. In under-saturated traffic, all vehicles, which are waiting within a queue at the signal, can pass the signal during the green phase.

Congested traffic at traffic signal is associated with *over-saturated city traffic*. In over-saturated traffic, some of the vehicles waiting within a queue at the signal cannot pass the signal during the green phase; as a result, the vehicle queue grows [5–20].

In classical theories, traffic breakdown, i.e., the transition from under- to over-saturated traffic at the signal should occur when the flow rate of vehicles q_{in} that arrive at traffic signal (called as the arrival flow rate or arrival traffic rate on the approach [19]) is larger than the classical signal capacity given by formula [5–20]

$$C_{\mathrm{cl}} = q_{\mathrm{sat}} T_{\mathrm{G}}^{(\mathrm{eff})}/\vartheta. \tag{1}$$

In (1), q_{sat} is the saturation flow rate, i.e., the mean flow rate from a moving vehicle queue occurring during green phase when vehicles discharge from the moving queue to their maximum free speed v_{free}; $\vartheta = T_{\mathrm{G}} + T_{\mathrm{Y}} + T_{\mathrm{R}}$ is the period (cycle time) of traffic signal that is assumed to be constant, T_{G}, T_{Y}, and T_{R} are durations of the green, yellow, and red phases of traffic signal, respectively; $T_{\mathrm{G}}^{(\mathrm{eff})}$ is an effective green phase time, which is the portion of the cycle time during which vehicles are assumed to pass traffic signal at constant rate q_{sat}. In empirical observations, a so-called lost time $\delta t = \vartheta - T_{\mathrm{R}} - T_{\mathrm{G}}^{(\mathrm{eff})} \approx 3$–4 s [19,20].[2]

As above-mentioned, in the classical theories of city traffic the signal capacity (1) determines the transition from under- to over-saturated traffic at the signal [5–20]. In contrast with the classical theories, as the author has found from an analysis of GW propagation through a single city intersection [21,22], at $\bar{q}_{\mathrm{in}} > C_{\mathrm{cl}}$ the GW can nevertheless persist during a random time interval as long as $\bar{q}_{\mathrm{in}} < C_{\mathrm{max}}$, where

$$\bar{q}_{\mathrm{in}} = \vartheta^{-1} \int_0^{\vartheta} q_{\mathrm{in}}(t)\mathrm{d}t \tag{2}$$

is the averaged arrival flow rate, C_{max} is a maximum signal capacity, which exceeds C_{cl}, i.e., $C_{\mathrm{max}} > C_{\mathrm{cl}}$.

While considering a model of GW propagation through a single intersection, we have found [21,22] that GW breakdown is initiated by the emergence of a moving synchronized flow pattern (MSP) within the GW. It has also turned out that MSPs can play an important role for the occurrence of traffic gridlock in city traffic [22,23]. However, physical features of MSPs in under-saturated city traffic have not been still understood. Moreover, a GW is a special case of city traffic. General conditions for traffic breakdown at the signal have not still been sufficiently studied.

To reveal general features of city traffic, in this article we develop three-phase theory for under-saturated city traffic.[3] The article is organized as follows. In Section 2, we discuss a stochastic microscopic three-phase model for city traffic used for all simulations, explain the basic assumption of this model, consider the physical meaning of model variables and parameters as well as study characteristics of MSPs and moving queues in the model. A model of a GW is discussed in Section 3. In Section 4, we study the effect of MSP characteristics on GW breakdown. The influence of turning-in traffic on characteristics of MSPs are studied in Section 5. Traffic phenomena caused by GW propagation though a sequence of city intersections are considered in Section 6. The probability of traffic breakdown for different time-dependences of the arrival flow rate and diagrams of traffic breakdown at the signal are studied in Section 7. A possibility of GW control leading to the prevention of GW breakdown is demonstrated in Section 8. In Section 9, we discuss qualitative differences of the effect of synchronized flow on the phenomenon *traffic breakdown* at traffic signal in city traffic and at an on-ramp bottleneck in highway traffic, discuss a breakdown minimization (BM) principle for optimization of city networks, compare classical theory of city traffic with three-phase traffic theory as well as formulate conclusions.

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[1] This has also been earlier assumed by the author (see Sec. 22.4 in Ref. [2] and footnote 1 in Chap. 1 of Ref. [3]).

[2] To explain the physical sense of the lost time, note that vehicles accelerate from a standstill within the queue at the signal location. At the beginning of the green phase, the flow rate at the signal location increases during some time interval from zero to q_{sat}. For this reason, in the definition of the classical signal capacity (1) some effective green phase time $T_{\mathrm{G}}^{(\mathrm{eff})}$ is used [19].

[3] In comparison with three-phase theory, in two-phase traffic flow models with free flow instability MSPs cannot occur. As found in Refs. [21–23], this can lead to considerably heavier conditions for traffic breakdown at the signal. A consideration of the associated two-phase traffic flow models is out of scope of this article.