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Metastability in the Hamiltonian mean field model and Kuramoto model

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Abstract

We briefly discuss the state of the art on the anomalous dynamics of the Hamiltonian mean field (HMF) model. We stress the important role of the initial conditions for understanding the microscopic nature of the intriguing metastable quasi-stationary states (QSS) observed in the model and the connections to Tsallis statistics and glassy dynamics. We also present new results on the existence of metastable states in the Kuramoto model and discuss the similarities with those found in the HMF model. The existence of metastability seems to be quite a common phenomenon in fully coupled systems, whose origin could be also interpreted as a dynamical mechanism preventing or hindering synchronization.

Keywords: Metastability; Coupled oscillators; Anomalous dynamics; Tsallis statistics; Glassy dynamics; Synchronization

1. Introduction

The Hamiltonian mean field (HMF) model intensively studied in the last years can be considered a paradigmatic system for understanding the behavior of long-range interacting systems [1]. In particular its striking out-of-equilibrium dynamics has raised much interest [2–15] for the anomalous dynamics connected with the existence of metastable quasi-stationary states (QSS). Several claims have been advanced concerning a theoretical description of this phenomenon within standard statistical mechanics [5–8]. However, these studies neglect the important role of the initial conditions and the hierarchical fractal-like structures which are generated in the μ-space. In our opinion the situation is still much more complex and not completely clear although some progress has certainly been made. In this very short paper we summarize our point of view based on the most recent numerical simulations [14] which indicate a connection to Tsallis generalized statistics [16] and glassy dynamics [13]. We present also another interesting point of view, which could probably help in understanding this anomalous behavior, exploiting an analogy with recent numerical results on metastability recently found in the Kuramoto model. In other words, metastability could likely be seen also as a kind of dynamical hindrance to synchronization.

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2. Metastability in the HMF model

The HMF model describes a system of planar rotators or spin vectors $\vec{s}_i = (\cos\theta_i, \sin\theta_i)$ with unitary mass and interacting through a long-range potential. The Hamiltonian can be written as

$$H = K + V = \sum_{i=1}^{N} \frac{p_i^2}{2} + \frac{1}{2N} \sum_{i,j=1}^{N} [1 - \cos(\theta_i - \theta_j)], \tag{1}$$

where θ_i and p_i are conjugate variables. The latter can also describe a system of fully interacting particles rotating on the unit circle. Introducing the *magnetization* $M = (1/N)|\sum_{i=1}^{N} \vec{s}_i|$ as order parameter, it is possible to solve the model exactly at equilibrium, where a second-order phase transition exists. The system passes from a low-energy condensed (ferromagnetic) phase with magnetization $M \neq 0$, to a high-energy one (paramagnetic), where spins are homogeneously oriented on the unit circle and $M = 0$. The critical point is found at a temperature $T_c = 0.5$, which corresponds to the critical energy density $U_c = 0.75$. The *caloric curve* is given by $U = H/N = T/2 + \frac{1}{2}(1 - M^2)$ [1].

At variance with this equilibrium scenario, the out-of-equilibrium dynamics—explored by means of numerical simulations—shows several anomalies in a special subcritical region of energy values and in particular for $0.68 < U < U_c$. Starting from out-of-equilibrium initial conditions, the system remains trapped in metastable QSS with vanishing magnetization at a temperature lower than the equilibrium one, until it slowly relaxes towards Boltzmann–Gibbs (BG) equilibrium. This transient QSS regime becomes stable if one takes the infinite size limit before the infinite time limit [2,3]. In this case the system stays forever in the $M = 0$ state at the limiting temperature T_{QSS}.

In several previous works [2,3] we have clearly shown that the microscopic nature of the anomalous QSS regime depends in a sensitive way on the choice of the initial conditions (IC). In this respect we focused on two main classes of IC (see Fig. 1).

(1) The first class of IC contains the so-called M1IC, with all the angles put equal to zero, i.e., $M(0) = 1$ and a *water bag* (uniform) distribution of momenta. In this class we can also consider those initial conditions with finite magnetization, $M(0) > 0$. When the system is started with these initial conditions an initial *thermal explosion*, with a following violent relaxation, occurs. After this stage, the system quickly freezes and remains trapped in long-living QSS with vanishing magnetization and a temperature smaller than the equilibrium one. In this case the system appears macroscopically quite homogeneous, since averaging over all angles the magnetization tends to vanish with the size of the system. On the other hand *from a microscopic point of view it is not so*. In fact fractal-like structures are generated in the μ-space and many dynamical anomalies appears, among which are anomalous diffusion, power-law decay of velocity correlations, aging, weak ergodicity breaking and dynamical frustration [2,3]. Such a behavior can be usefully described by the Tsallis' generalized formalism [14,16] and interesting analogies with glassy systems do also exist [13].

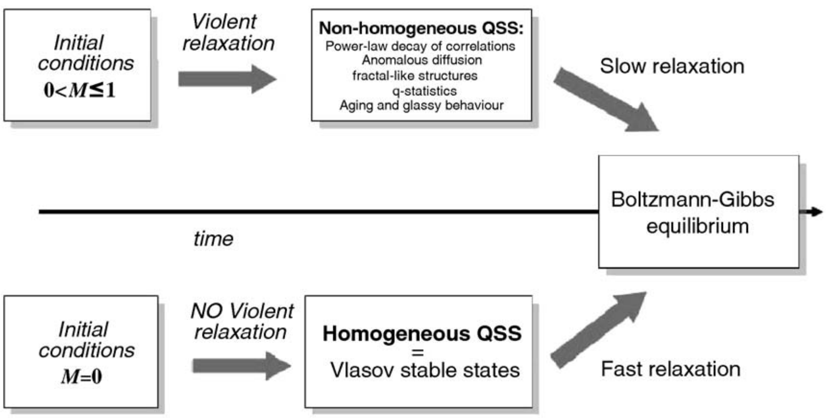

Fig. 1. Schematic figure illustrating the relaxation to BG equilibrium in the HMF model. According to the initial condition of the magnetization the nature of the QSS presents different features and the routes to thermalization are not the same. See text.