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The effects of mechanical response on the dynamics and string stability of a platoon of adaptive cruise control vehicles

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HIGHLIGHTS

- A new formalism treats the general mechanical response of adaptive cruise control vehicles.
- String stability of a platoon is necessary but not sufficient for a smooth ride.
- A secondary peak in the transfer function due to the delayed response can cause undesirable oscillations.
- Frequency-dependent acceleration feedback effectively eliminates oscillations.

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ABSTRACT

The dynamics of a platoon of adaptive cruise control vehicles is analyzed for a general mechanical response of the vehicle. Effects of acceleration-feedback control that were not previously studied are found. For small acceleration-feedback gain, which produces marginally string-stable behavior, the reduction of a disturbance (with increasing car number n) is found to be faster than for the maximum allowable gain. The asymptotic magnitude of a disturbance is shown to fall off as $erf\left(\frac{ct.}{\sqrt{n}}\right)$ when $n \rightarrow \infty$. For gain approaching the lower limit of stability, oscillations in acceleration associated with a secondary maximum in the transfer function (as a function of frequency) can occur. A frequency-dependent gain that reduces the secondary maximum, but does not affect the transfer function near zero frequency, is proposed. Performance is thereby improved by elimination of the undesirable oscillations while the rapid disturbance reduction is retained.

1. Introduction

The dynamics, in particular the string stability, of a platoon of adaptive cruise control (ACC) vehicles has been studied by numerous authors [1–11]. Interest in ACC vehicles is strong because they provide higher flow rates and have a lower probability to form jams than conventional vehicles driven by people. A major factor in the improved performance is the elimination of human reaction time. Likewise, mixing ACC vehicles in with manually driven vehicles has been shown to improve stability and increase traffic flow [12–24]. A mixture of the two types of vehicles with different stability characteristics shows modified phase transitions to congested or jammed states [16,21].

In recent years the effects of a time delay and a first-order time constant (associated with the vehicle's mechanical response) on the stability of ACC platoons have been discussed [25–29]. At low speeds the mechanical response can be slow enough to be a factor in stability. At higher speeds, the faster response is not so important unless quite small headways are desired, in which case the same considerations apply as at low speed. Physicists and mathematicians find the effect of

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delayed response on the dynamics of systems such as ACC platoons to be challenging and intriguing (see Refs. [25,27,29] for example). The purpose of the present work is to examine the effects of mechanical response more generally and in more detail than what has been reported previously.

In Section 2, a formalism is presented for calculating the dynamics of a platoon of ACC vehicles with a general response function that is not limited to a description based on an explicit delay and a first-order time constant. Section 3 contains simulations for a realistic response function modeled after the torque response to a step-function change in desired acceleration [30]. The rate at which a disturbance (such as that caused by a brief acceleration of the leading vehicle) decreases with increasing car number (from front to rear) is determined in Section 4. A frequency-dependent gain for acceleration-feedback control is proposed and evaluated in Section 5. Conclusions are drawn in Section 6.

2. General response

The control algorithm for the desired acceleration of each vehicle of a platoon is taken to be [26]

$$a^d(t) = \frac{\alpha}{h}[x_{\text{lead}}(t) - x(t) - D] - \alpha v(t) + k[v_{\text{lead}}(t) - v(t)] - \xi a(t). \quad (1)$$

The subscript "lead" designates the preceding vehicle and the parameters in Eq. (1) are the sensitivity α, the headway-time constant (also called gap-time constant) h, the coefficient of relative-velocity feedback k, and the acceleration-feedback gain ξ. D is the length of a vehicle plus a small safe distance.

If the desired acceleration were a step function of magnitude A

$$a^d(t) = 0, \quad t < 0 \quad (2a)$$

$$= A, \quad t \geq 0, \quad (2b)$$

the resulting acceleration would be $Ag(t)$, which has the properties

$$g(t) = 0, \quad t \leq 0 \quad (3)$$

and

$$g(t) \to 1, \quad t \to \infty. \quad (4)$$

For an arbitrary desired acceleration the acceleration is therefore

$$a(t) = \int_0^t \frac{\mathrm{d}g(t')}{\mathrm{d}t'} a^d(t - t')\mathrm{d}t'. \quad (5)$$

To determine the transfer function $G(\omega)$ in Fourier space let

$$\Lambda(\omega) = 1 - \mathrm{i}\omega \int_0^\infty \mathrm{e}^{-\mathrm{i}\omega t}[1 - g(t)]\mathrm{d}t. \quad (6)$$

Then the control algorithm Eq. (1) and the definition Eq. (6) give

$$G(\omega) = \frac{\frac{\alpha}{h} + \mathrm{i}k\omega}{\frac{\alpha}{h} - \omega^2\left[\xi + \Lambda(\omega)^{-1}\right] + \mathrm{i}\omega(\alpha + k)}. \quad (7)$$

The ACC model developed in this section is not complete because it does not consider limitations on acceleration or deceleration. Furthermore, the mechanical response differs for accelerating and braking and depends on engine speed (longer at low speeds), factors that must be accounted for in a realistic model applicable to all driving scenarios.

3. Simulations

To perform simulations I introduce discrete-time control for convenience. Let the desired acceleration now be given by

$$a^{\text{desired}}(t) = a_{\mathrm{d}\mu}, \quad \mu T \leq t < (\mu + 1)T. \quad (8)$$

T is the update time, typically 0.01 s and $a_{\mathrm{d}\mu} = a^d(\mu T)$ as given by Eq. (1). (μ is an integer).

The acceleration is then

$$a(t) = \sum_{m=0}^{\mu} a_{\mathrm{d}m}\left[g(t - mT) - g(t - mT - T)\right], \quad \mu T \leq t < (\mu + 1)T. \quad (9)$$

Suppose a sufficiently accurate description of the response function is given by

$$g(t) = g_\mu, \qquad \mu T \leq t < (\mu + 1)T, \quad 0 < \mu \leq N \quad (10a)$$

$$g(t) = 1, \quad t > NT, \quad (10b)$$