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Sales distribution of consumer electronics

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ABSTRACT

Using the uniform most powerful unbiased test, we observed the sales distribution of consumer electronics in Japan on a daily basis and report that it follows both a lognormal distribution and a power-law distribution and depends on the state of the market. We show that these switches occur quite often. The underlying sales dynamics found between both periods nicely matched a multiplicative process. However, even though the multiplicative term in the process displays a size-dependent relationship when a steady lognormal distribution holds, it shows a size-independent relationship when the power-law distribution holds. This difference in the underlying dynamics is responsible for the difference in the two observed distributions.

1. Introduction

Since Pareto pointed out in 1896 that the distribution of income exhibits a heavy-tailed structure [1], many papers has argued that such distributions can be found in a wide range of empirical data that describe not only economic phenomena but also biological, physical, ecological, sociological, and various man-made phenomena [2]. The list of the measurements of quantities whose distributions have been conjectured to obey such distributions includes firm sizes [3], city populations [4], frequency of unique words in a given novel [5,6], the biological genera by a number of species [7], scientists by a number of published papers [8], web files transmitted over the internet [9], book sales [10], and product market shares [11]. Along with these reports the argument over the exact distribution, whether these heavy-tailed distributions obey a lognormal distribution or a power-law distribution, has been repeated over many years as well [2]. In this paper we use the statistical techniques developed in this literature to clarify the sales distribution of consumer electronics.

To illustrate the heavy-tailed distribution's appearance, random growth processes are widely used as the approximation of its underlying dynamics. Gibrat, who built upon Kapteyn and Uven's work, was the first to propose the simplest form of this type of model, usually known as the multiplicative process, to describe the appearance of heavy-tailed distributions in firm size distributions [12]. His work is significant in market structure literature [13]. Even 70 years after Gibrat's book, more and more measures of quantities are found that are conjectured to obey this type of process.

Among recent works, Fu et al. [14] is crucial because it was perhaps the first work to consider the hierarchical structure of institutions. No one denies that firms grow in size and scope and that such a growth is heavily influenced by the successful launch of new products. Fu et al. modeled products as elementary sales units assuming that they evolve based on a random multiplicative process. They extended the usual model of proportional growth to illustrate the size variance relationship found in growth distributions at different levels of aggregation in the economy by considering hierarchical structure.

Many studies have analyzed product sales. Sornette et al. [10] used a book sales database from Amazon.com and performed a time series analysis of book sales by classifying endogenous and exogenous shocks. With a database of newspaper and magazine circulation, Picoli et al. [15] used the multiplicative process to illustrate the link between tent-shaped

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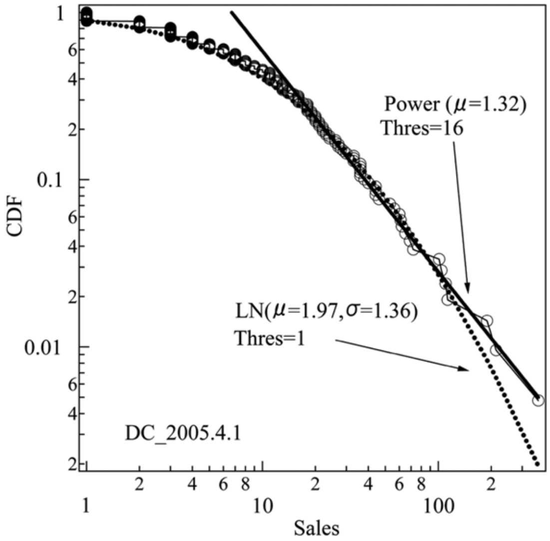

Fig. 1. Cumulative distribution of sales volume of digital cameras sold on April 1, 2005. Slashed line shows fitted maximum likelihood estimate assuming all points above 1 obeyed a lognormal distribution, and continuous line shows fitted maximum likelihood estimate assuming all points above 16 obeyed a power-law distribution. Parameters of both distributions are depicted as well.

log growth distributions and the power-law distributions found in the growth of newspaper and magazine sales. However the exact dynamics of product sales remains an open question.

In this paper we clarify the distribution and the dynamics of the sales of consumer electronics using a unique database of product sales from the Japanese consumer electronics market. The data were recorded daily, making it possible to track the actual sales volume of each product in a more detailed manner and to model the dynamics from a more empirically driven approach. We numerically analyzed more than 1200 sales distributions recorded on a daily basis from October 1, 2004 to February 29, 2008. Using the uniform most powerful test, we statistically show that sales distributions differ among different periods and occasionally exhibit a power-law behavior. We also show with the multiplicative process that the underlying ingredients of stochastic growth itself are different among these periods. Moreover, our findings are compatible with the mathematical results reported by Ishikawa et al. [16].

The paper is organized as follows. Section 2 provides an overview of our data set. Section 3 introduces the sales distribution of consumer electronics, and Section 4 illustrates our statistical technique regarding the verification of a true power-law distribution. Using this statistical technique, in Section 5, we show why the power-law behavior found in Section 3 can be considered as a genuine power-law behavior. Section 6 reports how sales distribution changes over time. Sales distribution exhibits both power-law and lognormal distributions. Section 7 focuses on the underlying dynamics of sales, providing another source of evidence that the dynamics of sales differs among different periods. Section 8 provides further discussion and a conclusion.

2. Sales data of consumer electronics

Consumer electronics chains sell products such as TVs, personal computers, audio devices, refrigerators, digital cameras, air conditioners, and DVD recorders. Their annual revenue amounts to 5.9 trillion yen in Japan. In this paper we investigate distribution using the sales data of digital cameras from 23 different consumer electronics chains in Japan collected by a private company called BCN Inc. This data set covers about 45% of all consumer electronics chains in Japan including over 1400 retail stores [17]. The data were recorded daily covering the period from October 1, 2004 to April 30, 2008.

3. Sales distribution of consumer electronics

We focused on the top selling products using cumulative distribution $P_>(S)$ defined as

$$P_>(x) := \Pr[X \geq x] = \int_x^{\infty} f(x')\mathrm{d}x' \quad (1)$$

where $f(x)$ describes the probability density function. The cumulative distribution of the sales volume of digital cameras on April 1, 2005 is shown on a double logarithmic scale (Fig. 1). It exhibits a heavy-tailed structure. To investigate the exact characteristic of its distribution, we also depict the maximum likelihood estimate of a lognormal distribution, assuming that all values above 1 obeyed a lognormal distribution and the maximum likelihood estimate of a power-law distribution, assuming that all values above 16 obeyed a power-law distribution. A lognormal distribution fits nicely for almost all points except the last three. For the points above 16 including these last three points, at first glance it seems that a power-law