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Class formation in a social network with asset exchange

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ABSTRACT

We study two kinds of economic exchanges, additive and multiplicative, in a system of N agents. The work is divided into two parts. In the first one, the agents are free to interact with each other. The system evolves to a Boltzmann–Gibbs distribution with additive exchange and condenses with a multiplicative one. If bankruptcy is introduced, both types of exchange lead to condensation. Condensation times have been studied. In the second part, the agents are placed in a social network. We analyze the behavior of wealth distributions in time, and the formation of economic classes is observed for certain values of network connectivity.

1. Introduction

There has been in the past few years a large amount of literature published dealing with the study of the distribution of wealth in agent based models with various kinds of interaction rules [1–3]. Several distributions, such as *Boltzmann–Gibbs*, *Gamma*, and *Pareto* ones, can be obtained, according to the different conditions of the models [4–6]. It is well known that real data analysis from several countries [7] yields a *Boltzmann–Gibbs* distribution for that sector of the population with the lowest wealth, who are the majority, and a *Pareto* distribution for the minority of the population with the highest values of wealth (see Fig. 1 for an example).

This particular behavior has been reproduced, to some extent, using different kinds of assumptions [8,9].

In this article we describe a model where this behavior is obtained and the appearance of social classes is observed. The model is agent based, with agents arranged into a social network, and we define very simple rules for wealth exchange. We analyze the conditions under which the different distribution types are obtained and also the conditions for the social classes to appear.

In the next section of the paper we describe the interaction model for several cases under study. We focus the analysis on the behavior of entropy, poverty (defined as the minimum of wealth, where the amount of money is less than the minimum allowed exchange) and wealth distributions in Sections 3 and 4.

In Section 3 the dynamics of the model, with and without bankruptcy (defined as the impossibility of participating in a trade due to the wealth being less than the minimum allowed exchange), is investigated. The agents are not placed in a network, which is equivalent to a system where the agents are placed in a fully connected network. In Section 4 we study the effect of a non-fully connected network. Finally in the last section we give our conclusions.

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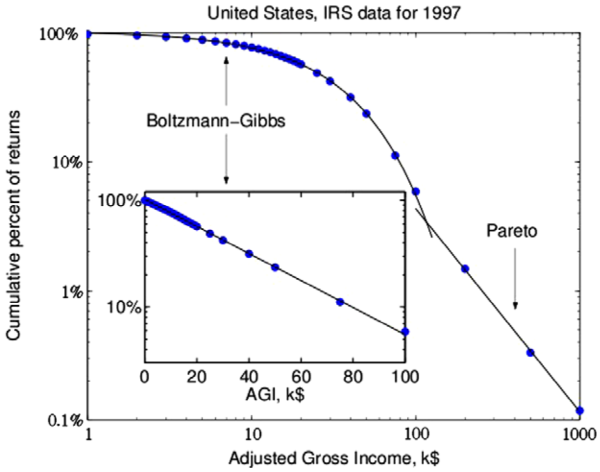

Fig. 1. Cumulative probability distribution of US individual income for 1997.
Source: Figure 6 from [1].

2. The interaction model

2.1. Wealth exchange models

The kinds of wealth exchange models that we have considered in the present work are those in which the total amount of wealth before and after the interaction is conserved, also known as elastic collision models. That is, if (i, j) are the labels of the two agents involved, and their wealths are (w_i, w_j) respectively, then we can write

$$w_i(t + \Delta t) = w_i(t) + \Delta w, \quad (1)$$

$$w_j(t + \Delta t) = w_j(t) - \Delta w. \quad (2)$$

Since wealth is preserved in the interaction, we have that $w_i(t+\Delta t)+w_j(t+\Delta t) = w_i(t)+w_j(t)$. It is important to remark that agents are not allowed to have negative wealth. Also is necessary to mention that agent i has a probability p of losing Δw, while agent j has a probability $1 - p$. In our model the agents have the same probabilities of winning and losing; then the value of p is 0.5. Furthermore the agents have only integer values of wealth; hence, Δw must be an integer value.

According to which value that we choose for Δw, we can have two distinct processes: additive or multiplicative exchange. In the additive exchange we have

$$\Delta w = \text{const},$$

and this means that the exchange money is fixed in time and is independent of the wealth [10]. In the multiplicative case, also known as the *yard-sale* model [11], we have taken Δw as

$$\Delta w = \text{round}(\nu \cdot \min(w_i(t), w_j(t))),$$

where we used round(x) as the integer closest to x, and $0 < \nu < 1$. One difference between the multiplicative case and the additive case is the dependence on time for Δw, which in the latter case is not shown.

2.2. The interaction model with no network, or an undirected fully connected network

We have considered a closed population composed of N agents, where each of them has the possibility of exchanging her wealth, through the exchange rules mentioned above, with any other agent in the population.

On the basis of the aforementioned, we can establish the equivalence between a population with no network and a population embedded in an undirected fully connected network, where each agent i has a connectivity Z^i [12] equal to $N - 1$. Therefore Eqs. (1) and (2) are valid for all (i, j) belonging to the population.

2.3. The interaction model with a undirected non-fully connected network

We introduce an undirected network in the population, in which every agent i has k_i links to other agents, such that $1 \leq k_i \leq k_{\max}$, where $k_{\max}$ is the highest possible number of links allowed in the dynamics. In other words we have used a random network with an arbitrary degree distribution such that $p(k)$ is given by

$$p(k) = \begin{cases} \dfrac{1}{k_{\max}}; & k \in [1, k_{\max}] \\ 0; & \text{otherwise.} \end{cases} \quad (3)$$