Physica A 389 (2010) 4105–4115

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Effect of speed fluctuations on a green-light path in a 2d traffic network controlled by signals

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ARTICLE INFO

Article history:
Received 29 January 2010
Received in revised form 25 May 2010
Available online 8 June 2010

Keywords:
Traffic dynamics
Green-light path
Fluctuation
Signal control
Complex system
Random walk

ABSTRACT

When a vehicle moves through a series of green lights, avoiding red signals in a two-dimensional (2d) city traffic network, the vehicle describes a characteristic trajectory (green-light path) and the travel time has a minimal value. The green-light path depends on the cycle time, split, signal-control strategy, and fluctuations of vehicular speed. We clarify the effect of speed fluctuations on a green-light path in a 2d traffic network controlled by signals. Even if an extremely small quantity of speed fluctuation is added, the green-light path changes greatly. It is shown that the root-mean square (RMS) of the deviation from the mean path depends highly on the cycle time. Also, the dependence of the green-light path on the speed-fluctuation strength is shown under a constant value of cycle time.

1. Introduction

Recently, vehicular traffic has attracted considerable attention [1–5]. The traffic flow, pedestrian flow, and bus-route problem have been studied from a point of view of statistical mechanics and nonlinear dynamics. Many observed dynamical phenomena in vehicular traffic have been successfully reproduced with physical methods [6–15].

In urban traffic, vehicles are controlled by traffic lights to give priority for a road and to insure the safety because they meet at crossings. The traffic characteristic depends highly on the control strategy of traffic lights. In real city traffic, the traffic lights are controlled by either synchronized or green-wave strategies. In the synchronized strategy, all the signals change simultaneously and periodically where the phase shift has the same value for all signals. In the green-wave strategy, the signal changes with a certain time delay between the signal phases of two successive intersections. The change of traffic lights propagates backward like a green wave.

One has studied the periodic traffic controlled by a few traffic lights [16,17]. Also, few works have been done for the traffic of vehicles moving through an infinite series of traffic lights with the same interval [18–22]. The operator is able to control the traffic signal by the use of the other strategy. The various methods of traffic control have been investigated [23,24]. The vehicular traffic depends highly on the signal's strategy only for a low density [25].

For city traffic, vehicles do not move on a single roadway but go on a two-dimensional traffic network controlled by traffic lights. When a driver selects a path in the 2d traffic network, the travel time depends on the selected path. If a driver wishes to move without a destination as soon as possible, he goes through a series of green-light signals. Then, the travel time has a minimal value. The vehicle draws a characteristic trajectory which depends on both the control strategy and the signal's characteristics. We call the trajectory the green-light path. In the previous letter [24], the dependence of the green-light path on the signal's characteristics has been clarified at the synchronized and random-phase strategies. Generally, the vehicular speed fluctuates when the vehicle goes through the city. The travel time varies with the speed's fluctuation when the vehicle moves between the traffic lights. By the difference of the travel time, the vehicle meets red or green lights at the crossing.

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0378-4371/$ – see front matter
doi:10.1016/j.physa.2010.05.050

The green-light path drawn by the vehicle changes with the time difference. Thus, the green-light paths will depend on the fluctuation of vehicular speed. However, little has been known how the green-light path depends on the speed fluctuation (noise). If one finds out the green-light path and his destination is close to the arrival point, he will arrive at his destination faster. Also, the green-light path is closely connected with the Brownian motion in the case of the random-phase strategy. Therefore, it is important and necessary to know the green-light path.

In this paper, we study the effect of speed fluctuations on the green-light path in the two-dimensional traffic network controlled by traffic lights. We present the stochastic model for the vehicular motion through the sequence of green lights by avoiding red signals. We clarify the dependence of green-light paths on both speed fluctuation and the signal's characteristics at the synchronized strategy.

2. Stochastic model

We consider the motion of a single vehicle going on a 2d city traffic network. The 2d traffic network is made of one-way perpendicular streets arranged on a square lattice with traffic lights where vertical streets are oriented upwards and horizontal streets are oriented rightwards. The traffic lights change from red (green) to green (red) with a fixed time period $(1-s_p)t_s$ $(s_p t_s)$. The period of green is $s_p t_s$ and the period of red is $(1-s_p)t_s$. Period t_s is called as the cycle time. Fraction s_p represents the split which indicates the ratio of the green time to the cycle time. All signals on vertical (horizontal) streets change simultaneously. The signals change alternately from horizontal streets to vertical streets.

If the driver wishes to go ahead as soon as possible, the minimal travel time is achieved by going ahead through a series of green lights. If the signal on a horizontal (vertical) street is red, the vehicle on the horizontal (vertical) street turns and goes ahead on the vertical (horizontal) street because the signal on the vertical (horizontal) street is green. Thus, the vehicle moves through the series of green lights by avoiding red signals. The vehicle draws a trajectory depending on the signal's characteristics and vehicular speed. We call the trajectory as the green-light path.

The traffic lights on a green-light path are numbered, from upstream to downstream, by $1, 2, 3, \ldots, n, n+1, \ldots$. The traffic lights are positioned with the same interval on the square lattice where the interval between signals n and $n+1$ is indicated by l. The Manhattan distance between the origin and signal n is given by nl. The vehicle moves with speed $v(n)$ between traffic light n and its next light $n+1$. The vehicular speed varies from position to position. Therefore, the travel time between two signals it takes to move fluctuates from signal to signal. The mean speed between two signals is given by such a value that the signal interval is divided by the travel time. When the mean speed varies from position to position randomly, it is uncorrelated with that at other positions. The speed is given by

$$v(n) = v_0 + v'(n), \tag{1}$$

where v_0 is the mean speed not depending on position, $v'(n)$ is the speed's fluctuation between signals n and $n+1$, $\langle v'(n)\rangle = 0$, and $\langle v'(n)v'(m)\rangle = \langle v'^2\rangle\delta_{nm}$. Here, $\delta_{nm} = 1$ for $n = m$ and $\delta_{nm} = 0$ for $n \neq m$.

We consider the synchronized strategy. In the synchronized strategy, all the traffic lights change simultaneously from red (green) to green (red) with a fixed time period $(1-s_p)t_s$ $(s_p t_s)$. All signals change periodically with period t_s.

The arrival time $t(n+1)$ at signal $n+1$ is given by $t(n+1) = t(n) + l/v(n)$. We use the dimensionless time $T(n) = \frac{t(n)v_0}{l}$ for time t. We use the dimensionless position $(X(n), Y(n))$ for position $(x(n), y(n))$ of the vehicle at signal n where $X(n) = x(n)/l$ and $Y(n) = y(n)/l$. The position of the vehicle at signal $n+1$ is given by

$$X(n+1) = X(n) + 1 - H\left(T(n) + T_{phase}(X(n), Y(n)) - T_s \,\mathrm{int}\left(\frac{T(n) + T_{phase}(X(n), Y(n))}{T_s}\right) - s_p T_s\right), \tag{2}$$

$$Y(n+1) = Y(n) + H\left(T(n) + T_{phase}(X(n), Y(n)) - T_s \,\mathrm{int}\left(\frac{T(n) + T_{phase}(X(n), Y(n))}{T_s}\right) - s_p T_s\right), \tag{3}$$

$$T(n+1) = T(n) + v_0/v(n), \tag{4}$$

where $T_{phase}(X(n), Y(n)) = t_{phase}(x(n), y(n))v_0/l$, and $T_s = t_s v_0/l$. $H(T)$ is the Heaviside function: $H(T) = 1$ for $T \geq 0$ and $H(T) = 0$ for $T < 0$. $H(T) = 1$ if the traffic light on the horizontal (vertical) street is red (green), while $H(T) = 0$ if the traffic light on the horizontal (vertical) street is green (red). All signals change periodically with period T_s.

The dimensionless cycle time increases linearly with the mean speed. Therefore, the effect of speedup corresponds to an increase of cycle time.

For the synchronized strategy, the phase shift is $T_{phase} = 0$ for all signals. If $s_p = 0.5$, Eqs. (2)–(4) reduces to the following

$$X(n+1) = X(n) + H(\sin(2\pi T(n)/T_s)), \tag{5}$$

$$Y(n+1) = Y(n) + 1 - H(\sin(2\pi T(n)/T_s)), \tag{6}$$

$$T(n+1) = T(n) + v_0/v(n). \tag{7}$$

Label n indicates the n-th signal on the green-light path. The distance from the origin to signal n is given by nl where l is the signal's interval. We normalized the Manhattan distance by interval l. The normalized Manhattan distance is given by n. Also, the vehicle moves without stopping at signals. The time it takes to move between a signal and its nearest-neighbor