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Formation and synchronization of autocatalytic noise-sustained structures under Poiseuille flow

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ABSTRACT

The formation and synchronization of 2D noise-sustained structures are investigated for Gray–Scott kinetics in packed-bed reactors under Poiseuille flows, when identical systems are submitted to independent spatiotemporal Gaussian white noise sources. A finite-wavelength instability is theoretically predicted and numerically confirmed for uncoupled reactors. In particular, noise-sustained structures that flow with viscous boundary conditions are numerically observed above threshold. When the systems are coupled in master–slave configuration, the numerical simulations show that the slave system replicates to a very high degree of precision the convective patterns arising in the master one due to the selective amplification of noise. The nature of the synchronization and the stability of the synchronization manifold are elucidated.

1. Introduction

Synchronization phenomena have been a topic of scientific research for many years [1,2] and in many systems, ranging from physics to biology [3], being ubiquitous in almost every scientific discipline. Since the most recent accounts on the status of this active field of pure and applied research [4–9] it has continued to grow at an explosive rate, incorporating new problems and perspectives, especially with regard to the synchronization of complex networks [10–12]. Many different situations have been considered, including synchronization of limit cycle oscillators [13–15], synchronization of chaotic systems [16], partial (i.e. phase) synchronization [17], generalized synchronization [18,19], synchronization of stochastic systems [20], etc. These works refer mainly to systems characterized by a purely temporal dynamics. A less explored field is the synchronization between continuous systems [21–25], in particular the synchronization of spatiotemporally chaotic fields [26–28] or stochastic fields [29–33].

In this paper we shall restrict our scope to non-delayed synchronization between stochastic fields. In particular, a topic that has been hardly addressed is the synchronization between noise-sustained structures (NSS) in systems undergoing a convective instability [32–34].

A convectively unstable regime is characterized by the fact that local perturbations to the steady state are advected more rapidly than their spreading rate [35–37]. When seen in a Lagrangian framework, the system is unstable; but from a Eulerian description – however – perturbations are "washed out by the flow". Macroscopic patterns named noise sustained structures (NSS) emerge in this regime if noise is present at all times. It is through dynamical amplification of random fluctuations that the system is driven out of its linearly unstable steady state towards the state sustaining NSS. Thus, if noise (or any external deterministic forcing) were not present, nonequilibrium structures could not arise. In fluid

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dynamics, for example, the NSS are a spatial macroscopic manifestation of amplified thermal fluctuations. NSS have been observed in fluid convection experiments (both in open flow configuration [38] and Taylor–Couette flows [39,40]), and their precursors have been also observed in nematic liquid crystals [41]. They have also been numerically shown to exist in optical systems [32,33,42–44] (driven in this case by quantum noise) and in an autocatalytic chemical reaction – the Gray–Scott (GS) model – taking place in a differential-flow reactor [45].

In previous works [34,45,46] we have investigated the synchronization of NSS for a cubic stochastic kinetics in extended chemical reactors, linearly coupled and submitted to a uniform external flow. The theoretical analysis reveals the existence of synchronization between the stochastic fields, but the results are restricted to the idealized situation of an unrealistic uniform flow. In this paper we extend that analysis to investigate the phenomena of pattern formation and synchronization of nonequilibrium structures for stochastic cubic kinetics, in extended reactors submitted to Poiseuille flow, i.e., under viscous boundary conditions that affect the shape and propagation of the resulting structures.

The paper is organized as follows: Section 2 reviews the equations for the isolated GS system under Poiseuille flow. The linear stability analysis of the steady state solution and the features of the resulting patterns are also discussed here. In Section 3 we illustrate the replication of NSS under unidirectional coupling and we characterize the stability of the synchronized manifold in terms of damped dynamics of Fourier modes. Finally, we summarize the main conclusions in Section 4.

2. Noise-sustained structures

2.1. The Gray–Scott kinetics

The GS model describes a three-step reaction, with the intermediate one having cubic autocatalytic kinetics:

$$P \overset{k_0}{\rightarrow} A$$
$$A + 2B \overset{k_1}{\rightarrow} 3B$$
$$B \overset{k_2}{\rightarrow} C.$$

In our case, the reaction is assumed to take place in a two-dimensional, differential-flow and packed-bed reactor, where A is immobilized while B is free to diffuse and is also advected by a flow. Moreover, the reaction is maintained out of equilibrium by keeping the concentration of the precursor species P constant ($p = p_0$) and that of the inert product C zero ($c = 0$), i.e. it is immediately removed from the reactor.

After scaling concentrations by $(k_2/k_1)^{1/2}$, time by k_2^{-1} and length by $(D_B/k_2)^{1/2}$, the rate equations for the system read

$$\begin{aligned} \partial_t a_1 &= \mu - a_1\, b_1^2 + \xi_1(\vec{r}, t), \\ \partial_t b_1 &= \nabla^2 b_1 - \vec{\phi}.\vec{\nabla} b_1 - b_1 + a_1\, b_1^2, \end{aligned} \tag{1}$$

where $a_1(\vec{r}, t)$ and $b_1(\vec{r}, t)$ are the local concentrations of A and B, respectively. μ stands for the scaled version of $k_0 p_0$ and the vector field $\vec{\phi}$ is that of a Poiseuille flow parallel to the longitudinal reactor axis (x direction), whose expression is

$$\vec{\phi}(\vec{r}) = \phi(y)\hat{x} = \frac{3}{2}\phi_m\left(1 - 4\frac{y^2}{L_y^2}\right)\hat{x}. \tag{2}$$

Here ϕ_m is the average fluid flux, and the flow velocity vanishes at the reactor walls $y = \pm L_y/2$ due to viscosity. Finally, $\xi_1(\vec{r}, t)$ in Eqs. (1) is a real Gaussian noise with intensity η_1, zero mean and δ-correlated in time and space, that accounts for local fluctuations in the external feeding μ.

2.2. Threshold analysis

We proceed to analyze the stability of the steady state solution under Poiseuille flow. Inspired by Ref. [47], we linearize Eqs. (1) around the uniform solution $a_1 = 1/\mu$ and $b_1 = \mu$, the eigenvalues $\omega(q_x)$ of the linear instability problem are obtained in terms of the wavevectors

$$\Delta a_1 = \tilde{a}(y)\exp(iq_x x + \omega t), \tag{3}$$

$$\Delta b_1 = \tilde{b}(y)\exp(iq_x x + \omega t), \tag{4}$$

through the following boundary eigenvalue problem

$$\begin{pmatrix} -\mu^2 & -2 \\ \mu^2 & 1 - q_x^2 - iq_x\phi(y) + \partial_{yy}^2 \end{pmatrix}\begin{pmatrix} \tilde{a} \\ \tilde{b} \end{pmatrix} = \omega\begin{pmatrix} \tilde{a} \\ \tilde{b} \end{pmatrix}. \tag{5}$$

This system was diagonalized numerically. In particular, a space discretization of the y-axis into N points was considered in order to get the eigenvalues ω. Note that by reflection symmetry around the x-axis, the original $2N \times 2N$ matrix (a square $N \times N$ matrix for each component) can be reduced to an $(N + 1) \times (N + 1)$ effective problem after an adequate discretization.