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Langevin simulation of scalar fields: Additive and multiplicative noises and lattice renormalization

N.C. Cassol-Seewald [a], R.L.S. Farias [b,*], E.S. Fraga [c], G. Krein [a], Rudnei O. Ramos [d]

[a] Instituto de Física Teórica, Universidade Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271 - Bloco II, 01140-070 São Paulo, SP, Brazil
[b] Departamento de Ciências Naturais, Universidade Federal de São João Del Rei, 36301-000 São João Del Rei, MG, Brazil
[c] Instituto de Física, Universidade Federal do Rio de Janeiro, 21941-972 Rio de Janeiro, RJ, Brazil
[d] Departamento de Física Teórica, Universidade do Estado do Rio de Janeiro, 20550-013 Rio de Janeiro, RJ, Brazil

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ABSTRACT

We consider the Langevin lattice dynamics for a spontaneously broken $\lambda\phi^4$ scalar field theory where both additive and multiplicative noise terms are incorporated. The lattice renormalization for the corresponding stochastic Ginzburg–Landau–Langevin and the subtleties related to the multiplicative noise are investigated.

1. Introduction

The relevance of mathematical methods of quantum field theory for statistical physics was recognized in the early seventies, particularly in the investigation of equilibrium phase transitions and critical phenomena. Renormalization theory, originally linked with the removal of infinities in perturbative calculations in quantum field theory, turned out to be a key element in the understanding of critical phenomena [1]. Functional integrals, Feynman diagrams and loop expansions are an integral part of the mathematical methods used presently in statistical physics — Ref. [2] is an excellent introductory text on concepts and methods of field theory in the quantum and statistical domains. Likewise, a large class of dynamic critical phenomena associated with time-dependent fluctuations about equilibrium states, naturally described in terms of stochastic partial differential equations [3,4], can also be formulated in terms of functional integrals and therefore are equivalently described by field theories [5]. Among the variety of phenomena associated with the dynamics of phase transitions, phase ordering [6] seems to be of particular importance for the understanding of the time scales governing the equilibration of systems driven out of equilibrium. The influence of the presence of an environment on the dynamics of particles and fields is encoded "macroscopically" in attributes that enter stochastic evolution equations, usually in the form of dissipation and noise terms. Relevant time scales for different stages of phase conversion can depend dramatically on the details of these attributes. Noise terms, in particular, introduce difficulties in the numerical simulation of the evolution equations; difficulties related to the well-known Rayleigh–Jeans catastrophe in classical field theory [7]. The present paper is concerned with the use of field-theoretic renormalization methods to control such catastrophe.

* Corresponding author. Tel.: +55 32 3373 4535; fax: +55 32 3379 2483.
E-mail addresses: nadiane@ift.unesp.br (N.C. Cassol-Seewald), ricardosonegofarias@gmail.com,ricardofarias@ufsj.edu.br (R.L.S. Farias), fraga@if.ufrj.br (E.S. Fraga), gkrein@ift.unesp.br (G. Krein), rudnei@uerj.br (R.O. Ramos).

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A typical dynamical equation describing phase conversion is of the form of a Ginzburg–Landau–Langevin (GLL) equation [8]:

$$\tau \frac{\partial^2 \varphi}{\partial t^2} - \nabla^2 \varphi + \eta \dot{\varphi} + \mathcal{V}'_{\text{eff}}(\varphi) = \xi(\mathbf{x}, t), \tag{1}$$

where φ is a real scalar field, function of time and space variables, $\mathcal{V}'_{\text{eff}}(\varphi)$ is the field derivative of a Ginzburg–Landau effective potential and η, which can be seen as a response coefficient that defines time scales for the system and encodes the intensity of dissipation, is usually taken to be a function of temperature only, $\eta = \eta(T)$. The function $\xi(\mathbf{x}, t)$ represents a stochastic (noise) force, assumed Gaussian and white, so that

$$\langle \xi(\mathbf{x}, t) \rangle_\xi = 0, \qquad \langle \xi(\mathbf{x}, t)\xi(\mathbf{x}', t') \rangle_\xi = 2\,\eta T \delta(\mathbf{x} - \mathbf{x}')\delta(t - t'), \tag{2}$$

in conformity with the fluctuation–dissipation theorem. The condensate $\langle \varphi \rangle_\xi$ plays the role of the order parameter, where $\langle \cdots \rangle_\xi$ means average over noise realizations. The second order time derivative in Eq. (1) appears naturally in relativistic field theories [9,10] or when causality is incorporated via memory functions [11,12] in the otherwise purely diffusive, first-order evolution equations [3,8]. However, a more complete, microscopic field-theory description of nonequilibrium dissipative dynamics [13] shows that the complete form for the effective GLL equation of motion can lead to much more complicated scenarios than the one described by Eq. (1), depending on the allowed interaction terms involving φ. In general, there will be nonlocal (non-Markovian) dissipation and colored noise, as well as the possibility of field-dependent (multiplicative) noise terms $\sim\varphi\,\xi$ accompanied by density-dependent dissipation terms. Another typical example is provided by stochastic Gross–Pitaevskii (SGP) equations [14,15] that incorporate thermal and quantum fluctuations of a Bose–Einstein condensate (BEC) on the traditional mean-field equation [16]. Such a stochastic equation can be derived from a microscopic inter-atomic Hamiltonian via the closed-time-path Schwinger–Keldysh effective action formalism [13]. In fact, on very general physical grounds, one expects that dissipation effects should depend on the local density $\sim\varphi^2\dot{\varphi}$ and, accordingly, the noise term should contain a multiplicative piece $\sim\varphi\xi$. The typical term coming from fluctuations in the equation of motion for φ will be a functional of the form [13]

$$\mathcal{F}[\varphi(x)] = \varphi(x) \int d^4x' \varphi^2(x') K_1(x, x') + \int d^4x' \varphi(x') K_2(x, x'), \tag{3}$$

where $K_1(x, x')$ and $K_2(x, x')$ are nonlocal kernels expressed in terms of retarded Green's functions and whose explicit form depends on the detailed nature of the interactions involving φ. Explicit treatments for these nonlocal kernels show that under appropriate conditions one is justified to express the effective equation of motion for φ in a local form (see e.g. Ref. [17] and references therein). The existence of these additional terms in the GLL equation will, of course, play an important role in the dynamics of the formation of condensates. For instance, it was shown that the effects of multiplicative noise are rather significant in the Kibble–Zurek scenario of defect formation in one spatial dimension [18].

Although in the literature there are many different approaches for studying the nonequilibrium dynamics in field theory [19–21], the use of stochastic Langevin-like equations of motion is still a simpler and more direct approach in many different contexts in statistical physics and field theory in general. For example, some of us have considered the effects of dissipation in the scenario of explosive spinodal decomposition: the rapid growth of unstable modes following a quench into the two-phase region of the quantum chromodynamics (QCD) chiral transition in the simplest fashion [22]. Using a phenomenological Langevin description inspired by microscopic nonequilibrium field theory results [10,23,24], the time evolution of the order parameter in a chiral effective model [25] was investigated. Real-time $(3 + 1)$-dimensional lattice simulations for the behavior of the inhomogeneous chiral fields were performed, and it was shown that the effects of dissipation could be dramatic in spite of the very conservative assumptions that were made. Later, analogous but even stronger effects were obtained in the case of the deconfining transition of $SU(2)$ pure gauge theories using the same approach [26,27]. Recent work in these directions by a different group can be found in Refs. [28,29].

In the present paper we consider the nonequilibrium dynamics of the formation of a condensate in a spontaneously broken $\lambda\varphi^4$ scalar field theory within an improved Langevin framework which includes the effects of multiplicative noise and density-dependent dissipation terms. The corresponding stochastic GLL equation can be thought of as a generalization of the results of Ref. [10] to the case of broken symmetry. The time evolution for the formation of the condensate, under the influence of additive and multiplicative noise terms, is solved numerically on a $(3 + 1)$-dimensional lattice. Particular attention is devoted to the renormalization of the stochastic GLL equation in order to obtain lattice-independent equilibrium results.

The paper is organized as follows. In Section 2 the proper lattice renormalization of the GLL, in order to achieve equilibrium solutions that are independent of lattice spacing, is addressed. In Section 3 the question of time discretization for a GLL with multiplicative noise is discussed. In Section 4 we show the results of our lattice simulations to study the behavior of the condensate. Section 5 contains our conclusions and perspectives.

2. Stochastic GLL equations

In our study, we consider an extended GLL equation, incorporating additive *and* multiplicative noise terms. The time evolution of the field $\varphi(\mathbf{x}, t)$ at each point in space, which will determine the approach of the condensate $\langle \varphi \rangle$ to its