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Tolerance-based punishment in continuous public goods game

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ABSTRACT

Altruistic punishment for defectors is considered as a key motive for the explanation of cooperation. However, there is no clear border between the cooperative and defective behaviors in a continuous strategy game. We propose a model to study the effect of punishment on the evolution of cooperation in continuous public goods game, wherein individuals have the traits to punish the co-players based on social tolerance. We show that a reasonable punishment with a uniform tolerance can spur individuals to make more investments. Additionally, for a fixed punishment cost and a fixed fine, a moderate value of tolerance can result in the best promotion of cooperation. Furthermore, we investigate the coevolutionary dynamics of investment and tolerance. We find that the population splits into two branches: high-tolerance individuals who make high investments and low-tolerance individuals who make low investments. A dynamic equilibrium is achieved between these two types of individuals. Our work extends punishment to continuous cooperative behaviors and the results may enhance the understanding of altruistic punishment in the evolution of human cooperation.

1. Introduction

The origin and maintenance of cooperation has been a challenging problem in evolutionary biology since Darwin [1–5]. Cooperation means behaviors that incur a cost to the individual who performs it, whereas benefits the other individual or individuals. Thus, the selfish individuals, who do not cooperate, will have a fitness advantage over cooperative individuals, since they can reap the benefits of cooperation without bearing the costs. Under natural selection, cooperation will vanish, which is inconsistent with the observation cooperation is omnipresent [6,7].

The public goods game (*PGG*) has been widely employed to investigate the evolution of cooperation among self-regarding individuals with the framework of evolutionary game theory [8,9]. The traditional *PGG* is a discrete-choice model, where individuals choose either to cooperate or defect, mapping full cooperation or nothing at all respectively. However, behaviors in real systems can hardly be expected to have this dramatically discrete nature. Indeed there is considerable evidence that cooperative behavior in nature should be viewed as a continuous rather than a discrete trait [10–12]. Therefore, the study of cooperative behavior in a quantitative way rather than a qualitative way is crucial in dealing with the problem of evolution and the stability of cooperation. A number of papers studying games among individuals with variable degrees of cooperation have recently appeared [12–18]. Wahl and Nowak examined a version of the continuous prisoner's dilemma game with a continuous three-dimensional strategy space [14]. Based on a combination of computer

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simulation and analytical results, they concluded that cooperative strategies that resisted invasion had the characteristics of being optimistic, generous and uncompromising. However, cooperation in the continuous prisoner's dilemma game with noise was found to be evolutionarily unstable [15]. Killingback et al. introduced a model of cooperation which is based on the concept of investment, and they showed that cooperation easily evolves from a non-cooperative initial state, and is sustained at relatively high levels [12]. The continuous game on spatial structures have also been addressed [18,19]. A complete review of previous researches on the evolution of cooperation in variable-investment systems is provided in Refs. [20,21].

Costly punishment, refers to an action that implies a fine on the punished person and the punisher also pays a cost, has been identified as one possible route to cooperation [22–26]. If those who free ride on the cooperation of others (defectors) are punished, cooperation may prevail. Yet this 'solution' begs the question of who will bear the cost of punishing the free riders on cooperation. Indeed, in previous research, individuals are divided into two types: punishing and nonpunishing, standing for people who will punish the free riders and who will not, respectively. With this hypothesis, punishing strategies lose the evolutionary competition in case of well-mixed interactions. However, if individuals have spatial neighborhood interactions, punishment can sustain cooperation [26]. Differently with the above researches, where punishment is always targeted at non-cooperators, Rand et al. relax this assumption and study the effect of so-called 'anti-social punishment' [27,28]. They found that when 'anti-social punishment' is possible, costly punishment no longer promotes cooperation.

Considering the continuous strategies in the public goods game, where no distinct demarcation between cooperator and defector exists, we propose an extended model with punishment based on individuals' social tolerances [29,30]. Each individual has the trait to punish the co-player whose cooperative level is beyond his social tolerance degree. In this study, we aim to explore how punishment based on tolerance influences cooperative behavior in evolutionary continuous public goods game. By using Monte Carlo simulations we demonstrate that reasonable punishment can enhance the cooperation. Moreover, with fixed punishment cost and fine, moderate values of tolerance range can result in the best environment for the viability of cooperators. We also investigate the coevolutionary dynamics of individuals' investments and tolerances. The rest of this paper is organized as follows. In Section 2, we give a brief introduction to our evolutionary game model. In Section 3, the simulation results and discussion are provided. Finally, concluding remarks are drawn in Section 4.

2. Model

We consider the evolutionary continuous public goods game on a periodic square lattice, where all the nodes have four neighbors. Each site on the lattice is occupied by an individual, represented by the index i. Every individual participates in $M = 5$ groups centered on his neighbors and himself respectively. The degree of cooperation of an individual i is characterized by its investment (strategy) $s_i(0 \leq s_i \leq 1)$. Individual i contributes an investment s_i to each group he participates in. Subsequently, the sum of investments in a group is multiplied by the 'synergy factor' r, and the resulting amount is then shared equally among all members of the group, irrespective of their strategy. Thus, the payoff i obtained from one group is $r \cdot \sum_{j\in\Omega} s_j/5 - s_i$ where Ω includes all the members in that group. The overall payoff individual i collected from all the groups he belongs to is represented as P_i, if no punishment is applied. Otherwise, the overall payoff, P_i, quantifying the 'fitness' of player i is obtained by subtracting punishment costs and/or punishment fines. The punishment yields $P_i' = P_i - \sum n_p \cdot \beta - \sum n_d \cdot \alpha$, where the sum runs again over all the groups containing i. n_p is given by the number of individuals who determine to punish i in each group contained i, and n_d is the number of individuals i will punish in each group involved. Punishment implies that the punisher pays a cost α, and the punished incurs a fine β. Here, individuals are assumed have the trait to punish the partners whose investments are beyond their social tolerances. The social tolerance of each individual, i, is denoted by a parameter $h_i(0 < h_i < 1)$. Thus, i tends to punish the members (denoted by j) of each group involved, if $s_j < s_i - h_i$. To update the strategy of players, we employ the Monte Carlo simulation procedure. At the end of each round, i selects a role model, j, from his direct neighbors randomly. Following the determination of payoffs P_i' and P_j', as described above, individual i imitates the strategy, s_j, of player j with probability

$$W = \frac{1}{1 + \exp[(P_i' - P_j')/\kappa]}, \tag{1}$$

where κ characterizes the intensity of selection [31]. For $\kappa \to 0$, i will adopt j's strategy if and only if $P_j' > P_i'$. In most cases, selfish and rational individuals here prefer the strategies of more successful neighbors. In line with most previous studies, we set κ to be 0.1 (strong selection). Particularly, some noise in the form of a small mutation is introduced in the imitation. If i will adopt j's strategy in the next generation, then $s_i(t + 1) = s_j(t) + \delta$, where δ is a random number from the interval: $(-0.005, +0.005)$.

At first, we assume that each individual has the same tolerance threshold (h). A bigger h indicates a more tolerant circumstance. For $h = 1$, no punishment occurs among individuals, in that even a full cooperator can tolerate a total free-rider. Then, an individual's investment and tolerance are combined as his strategy and the coevolutionary dynamics are investigated. Additionally, the effect of parameter r on the game dynamics has been addressed in many works. In our present study, we simply set it as 4.0 and focus our attention on the effect of punishment and tolerance on the evolution of