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Influence of the variance of degree distributions on the evolution of cooperation in complex networks

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ABSTRACT

We study how initial network structure affects the evolution of cooperation in a spatial prisoner's dilemma game. The network structure is characterized by various statistical properties. Among those properties, we focus on the variance of the degree distribution, and inquire how it affects the evolution of cooperation by three methods of imitation. For every method, it was found that a scale-free network does not always promote the evolution of cooperation, and that there exists an appropriate value of the variance, at which cooperation is optimal.

1. Introduction

How to explain the fact that altruistic behavior exists in the real world has drawn considerable attention in many fields including sociology, economics, and biology. Simulation of the prisoner's dilemma (PD) is a powerful method for the analysis of cooperative action [1]. There are several models of PD, in particular, the spatial prisoner's dilemma (SPD) which is capable of modeling interaction among individuals, has been widely explored. In the PD game, two players simultaneously decide whether to cooperate or defect. Both individuals receive a payoff R under mutual cooperation and P under mutual defection. A cooperator receives S when playing with a defector, who in turn receives T, with $T > R > P > S$. As a result, during a single round of the PD game it is obviously better to defect, regardless of the opponent's strategy. If every individual interacts with all other individuals, cooperators are unable to resist invasion by defectors, whereas under replicator dynamics, evolution of cooperation takes place in a well-mixed population [2].

Since the pioneering SPD study which investigated how lattices affect the evolution of cooperation [3], the effects of several types of spatial structures, such as regular random graphs [3], small-world networks [4] and scale-free (SF) networks [5–7], have been investigated. Santos et al. compared the evolution of cooperation in regular ring graphs with that for SF networks generated via the mechanisms of growth and preferential attachment. They showed that cooperation becomes the dominating trait in SF networks [5]. Then various statistical properties of networks have been recognized as key aspects of appropriately characterizing complex networks, such as cluster coefficients [8], degree correlation [9] and

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community structure [10]. Moreover, interactions among these statistical properties have been investigated recently [11,12]. As another approach to studying the influence of network structure, the evolution of cooperation for real social networks has been shown [11,13]. In addition, other studies have considered a model of co-evolution relating the evolution of cooperation and the network structure [14–16]. After Santos et al. argued for the influence of a SF network, most studies have adopted a SF network for SPD studies. However, they only considered degree distribution for a particular type of SF network, and they did not investigate the effect across different SF networks or power exponents. Fu et al. shows the network which have too bigger degree variance suppress the evolution of cooperation [17]. In addition, there are several different methods of imitation as in Refs. [5,11,14,18,19] and the differences between these methods are not clear.

In this paper, we investigate the effects of different SF networks generated with various power exponents on SPD. The power law is dependent on the degree distribution, $P(k) \sim k^{-\gamma}$. In particular, we focus on the normalized degree variance [14] for characterizing each network. Moreover, we adopt three methods of strategy imitation, the wealthiest imitation [14], the random select imitation [5] and the roulette select imitation. Our investigation shows that for every method of imitation, there exists an appropriate value of the variance, at which cooperation is optimal.

2. The model

On networks of N nodes created by the following algorithm, the replicator dynamics is implemented similarly as in Refs. [5,14]. At first, each individual i occupies a node of the constructed network and has the same probability of choosing cooperation or defection as an initial strategy. According to the payoffs, $R = 1, P = S = 0, T = b\ (>1)$, b, which denotes the temptation of defection is the only parameter, so each individual i $(1 \le i \le N)$ plays PD games with all its neighbors V_i and accumulates the resulting payoff π_i at each step. Then, all individuals i synchronously update their strategy. They use three methods of imitation. (a) The first is the wealthiest imitation discussed in Ref. [14], which is such that each individual i imitates the strategy of the wealthiest among its neighbors V_i. If an individual i has the highest payoff among the V_i, it retains its own strategy for the next step. (b) The second method is the random select imitation discussed in Ref. [5]. Each individual i chooses one of its neighbors at random, say j, and compares their respective payoffs π_i and π_j. If the neighbor's payoff is lower or equal, $\pi_i \ge \pi_j$, the individual i retains its strategy. On the contrary, if the neighbor's payoff is higher, $\pi_i \le \pi_j$, i imitates j's strategy with probability $(\pi_j - \pi_i)/[b * k_>]$ where $k_>$ is the highest degree between i and j. (c) The third is the roulette select imitation discussed in Ref. [20]. In this method, each individual i chooses one of its neighbor j and imitates j's strategy with probability $p_i(j) = \pi_j/(\pi_i + \sum_j \pi_j)$, or it retains its strategy with probability $p_i(i) = \pi_i/(\pi_i + \sum_j \pi_j)$.

To create different networks with various power exponents, we adopt the configuration model proposed by Newman [21] which can create a network with an arbitrary degree distribution. In order to keep the number of nodes and links of networks constant, we adopt the following algorithm. First, set the value of the minimum degree as $k_{min} = 1$ and assign stubs of links to N nodes according to the predetermined power law distribution with exponent Γ. If there exist nodes (isolated nodes) that have no link owing to strict reconstruction of the power law distribution, one stub of the link is added to each isolated node. These additional stubs affect the power law degree distribution only slightly because the proportion of nodes of minimum degree is large in the power law degree distribution. Then, to keep the number of links as M (or the average degree $\langle k\rangle = 2M/N$), add stubs of links to each node until the total number of stubs becomes $2M$. That is, we change the value of the minimal degree k_{min}. In this study, the first number of stubs never exceeds $2M$; Γ lies in the range $(1.6 \le \Gamma \le 2.9)$. Lastly, pick stubs in pairs of different nodes at random and connect them to create links unless the link is self-connected or a link between selected nodes already exists. Repeat this operation until no stub exists. If there are stubs for which we cannot build a link (typically there is no edge left, and there are seldom more than one or two edges left), we substitute for them by randomly connecting nodes. Thus we can obtain networks with constant numbers of nodes and links, closely approximating a power law distribution.

For characterizing constructed networks, we focus on the normalized degree variance $\sigma_n^2 = (\langle k^2\rangle - \langle k\rangle^2)/\langle k\rangle$ [14], where $\langle k\rangle$ is the average degree. An important property of σ_n^2 is that a random network of a Poisson distribution has $\sigma_n^2 \risingdotseq 1$. In addition, several researchers have used this variable to represent the heterogeneity of complex networks [22,23]. In studies on the SPD, Zimmermann et al. [14] and Fu et al. [17] used this variable to analyze their results. In this paper, the normalized degree variance is used to represent the heterogeneity and to compare the results obtained in previous studies.

The normalized degree variance σ_n^2 obtained from the constructed network is plotted with respect to the power law exponent Γ of the predetermined distribution in Fig. 1. The relationship between the power law exponent Γ of the predetermined distribution and the power law exponent γ estimated from the resultant distribution of our algorithm is shown in Fig. 2. Since the constructed networks by our algorithm does not sufficiently realize the degree distributions $P(k) = ck^{-\gamma}$ as shown below, the value of γ does not coincide with Γ.

A typical set of degree distributions for different values of the normalized variance is shown in Fig. 3. The distributions are approximately scale free. However, for small normalized variances, the distributions in the low degree region do not follow the power law, and heavy tails appear in the high degree region. In addition, a network characterized by large σ_n^2 has several hubs, whereas there are few large hubs in a network of small σ_n^2, which is more similar to a homogeneous network. Here we remark that the finiteness of the network is crucial in our analysis. The reason is described as follows. Let the degree distribution $P(k)$ of the constructed network exactly follow $P(k) = ck^{-\gamma}$, where $c = 1/\sum_{k=k_{min}}^{k_{max}} k^{-\gamma}$, and k_{max} denotes the