Physica A 388 (2009) 4364–4378

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Extremum complexity in the monodimensional ideal gas: The piecewise uniform density distribution approximation

Xavier Calbet [a,*], Ricardo López-Ruiz [b,a]

[a] BIFI, Universidad de Zaragoza, E-50009 Zaragoza, Spain

[b] DIIS, Universidad de Zaragoza, E-50009 Zaragoza, Spain

ARTICLE INFO

Article history:
Received 20 February 2009
Received in revised form 8 June 2009
Available online 2 July 2009

PACS:
89.75.Fb
05.45.-a
02.50.-r
05.70.-a

Keywords:
Nonequilibrium systems
Ideal gas
Complexity

ABSTRACT

The extremum complexity distribution is shown to be equivalent to a piecewise uniform distribution in the accessible N-dimensional phase space of a dynamical system. This leads to piecewise exponential functions as one-particle distribution functions. It seems plausible to use these distributions in some systems out of equilibrium, thus greatly simplifying their description. In particular, an isolated ideal monodimensional gas far from equilibrium follows two non-overlapping Gaussian distribution functions. This is demonstrated by numerical simulations. Also, some previous laboratory experiments with granular systems display this kind of distribution.

1. Introduction

In general, a variational formulation can be established for the principles that govern the physical world. Thus, the technique of extremizing a particular physical quantity has been traditionally very useful for solving many different problems. A notable example in the thermodynamics field is that of the maximum entropy principle, which basically states that a system under constraints (i.e. isolated) will evolve by monotonically increasing its entropy with time and will reach equilibrium at its maximum achievable entropy [1]. This principle is valuable in two distinct ways. First, it unambiguously provides a way to determine the state of equilibrium, which for the case of an isolated system will be that of the equiprobability among the accessible states. This property is useful within the field of equilibrium thermodynamics or thermostatics. Secondly, it gives a definite direction in which the system will evolve toward equilibrium, which is effectively an arrow of time. This property is valuable by restricting the evolution of systems to those of ever growing entropy. It also tells us that the entropy is equivalent to a stretched or compressed time axis. In summary, the latter property is useful for thermodynamics in its broader sense, that is, for systems out of equilibrium.

Recently [2], the extremum complexity assumption has been proven valuable for greatly restricting the possible accessible states of an isolated system far away from equilibrium. It states that isolated systems out of equilibrium can be simplified by assuming equiprobability among some of the total accessible states and zero probability of occupation for the rest of them. Equivalently, we can say that the probability density function of the system is approximated by a piecewise uniform distribution among the accessible states. The spirit of this hypothesis is that in some isolated systems local complexity can arise despite its increase in entropy. A typical example being life which can be maintained in an isolated system as long as internal resources last.

* Corresponding author.
E-mail addresses: xcalbet@googlemail.com (X. Calbet), rilopez@unizar.es (R. López-Ruiz).

0378-4371/$ – see front matter
doi:10.1016/j.physa.2009.06.049

In this paper, we will justify this hypothesis and will extend this concept applied to the monodimensional ideal gas. In this paper an ideal gas is to be understood as a theoretical gas composed of a set of randomly-moving point particles that interact only through elastic collisions. It will be shown that for some isolated systems relaxing towards equilibrium, it is a good approximation to assume that the system follows a series of states with extremum complexity, the *extremum complexity path*. The usefulness of this idea resides in simplifying the dynamics of the system by allowing us to describe very complex systems with just a few parameters. Advancing some of our results, in Section 6, the state of a monodimensional ideal gas far from equilibrium with 10,000 particles will be explained by a reduced set of only nine variables. We shall also see in Section 8 how some experiments with granular systems [3] also seem to show an extremum complexity distribution.

In Section 2, the equivalence between extremum complexity states and piecewise uniform distributions will be presented. A justification for this assumption will be explained in Section 3. These concepts will be applied to the monodimensional ideal gas in Section 4. The extremum complexity distribution and approximations in this monodimensional ideal gas are shown in Sections 5 and 6, where the assumption of extremum complexity will be shown to greatly simplify the dynamics of the system. Results of the numerical simulation of the monodimensional ideal gas are presented in Section 7. Some distributions found in experiments with granular systems [3] also seem to be extremum complexity ones. This is suggested in Section 8. Finally, a discussion of the results is given in Section 9.

2. Extremum complexity distribution is a piecewise uniform density distribution

The concept of what has been called LMC complexity [4] is currently being applied in many areas of physics, for example [11] where it is applied to an atom. This definition of complexity is intuitively based on the notion that a perfectly ordered crystal and a completely random gas should exhibit a very low or null complexity, but their range of entropy goes from zero, for the crystal, to a maximum one for the gas. We can now define a quantity that behaves in the opposite sense, disequilibrium, which would be the distance of the system to the equiprobability. This parameter would be zero for the gas and would have a high value for the crystal. Defining complexity as the product of these two quantities would give us the expected intermediate values for more general systems (see Ref. [4] for a more complete description).

When an isolated system relaxes towards equilibrium, it does so by monotonically increasing its entropy. In this context, the entropy is equivalent to a stretched time axis. In some particular cases [2,5], the extremum complexity hypothesis can also be assumed, which means that we have a supplementary constraint, namely, the isolated system prefers to relax toward equilibrium by approaching or following the extremum complexity path [5]. Let us formulate all this quantitatively.

The LMC complexity, C, given by [4], is defined as

$$C = H \cdot D, \tag{1}$$

where the disequilibrium, D is defined in Ref. [4] as the distance of the system state to the microcanonical equilibrium distribution, the equiprobability,

$$D = \sum_{i=1}^{N} (f_i - 1/N)^2, \tag{2}$$

and H is the normalized entropy,

$$H = -(1/\ln N) \sum_{i=1}^{N} f_i \ln f_i, \tag{3}$$

where N is the number of accessible states and f_i, with $i = 1, 2, \ldots N$, is the probability of occurrence of a particular state i of the system. Other authors have proposed different definitions for the disequilibrium, which are claimed to exhibit a more appropriate behavior than the original definition for some particular applications. See Martin et al. [6] for a comprehensive list of them. In any case, the extremum complexity distribution happens to be identical for almost all of these LMC-like complexities [6].

The extremum complexity distribution can be calculated by finding the complexity extrema for a given entropy, H, using Lagrange multipliers [5]. Table 1 shows the resulting distribution functions. The extremum distribution function is graphically shown for all accessible states of the system in Fig. 1. It can be subdivided in two components, one with the maximum probability, which will be referred as "king distribution", and another one with the rest of the non-zero probabilities, which will be named "people distribution". An important aspect to note is that both distributions are uniform within a certain domain of the phase space and zero everywhere else. They are effectively piecewise uniform distributions. In this respect, extremum complexity distributions are equivalent to piecewise uniform ones.

3. Evolution of a piecewise uniform distribution

How does a system evolve if it is initially in a piecewise uniform distribution? The evolution of the system between this initial state and the equilibrium one will obviously depend on the particular example studied. Still, we will show that, for some systems, a good approximation is to assume that the system evolves from one piecewise uniform distribution to another piecewise uniform distribution until it eventually reaches the equilibrium, another uniform distribution,