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Visibility graph approach to exchange rate series

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ABSTRACT

By means of a visibility graph, we investigate six important exchange rate series. It is found that the series convert into scale-free and hierarchically structured networks. The relationship between the scaling exponents of the degree distributions and the Hurst exponents obeys the analytical prediction for fractal Brownian motions. The visibility graph can be used to obtain reliable values of Hurst exponents of the series. The characteristics are explained by using the multifractal structures of the series. The exchange rate of EURO to Japanese Yen is widely used to evaluate risk and to estimate trends in speculative investments. Interestingly, the hierarchies of the visibility graphs for the exchange rate series of these two currencies are significantly weak compared with that of the other series.

1. Introduction

Time series analysis attracts persistent attention due to its potential use in theory and practice in diverse research fields [1]. Theories in physics are one of the most important origin's of the ideas and methods. The concepts in nonlinear physics have led great achievements in time series analysis, such as the long-range correlations, scale-invariance, chaotic property, and complexity (see, for instance, Refs. [2–6] and the references therein).

Complex network theory [7] is a new branch in statistical physics, in which complex systems are described with networks. The nodes and edges represent the elements and the relationships between them, respectively. The goal is to understand, in a global way, the impacts of topological structures on dynamics and functions. Hence, a natural question arises, namely, how can we use the network theory in time series analysis?

Recently, several efforts have been made to bridge time series and complex networks [8–14]. Zhang et al. [8–11] firstly considered pseudo-periodic time series. For an oscillatory time series, one can extract the cycles denoted with $\{C_1, C_2, \ldots, C_m\}$. Then we can map each cycle to a node and link the nodes whose corresponding cycles are morphologically similar (measured quantitatively by phase space distance or correlation coefficient). It is found that noisy periodic signals generate random networks, while chaotic time series generate networks exhibiting small-world and scale-free features. What is more, time series generated by different types of continuous dynamics, including periodic, chaotic, and periodic with noise, have distinctive local patterns.

Alternatively, Lacasa et al. [12,13] proposed the so-called visibility graph, in which the successive scalar time series points are mapped to nodes and each node is connected with all the other nodes covered within its visual line. The constructed networks inherit several properties of the time series. To cite examples, periodic, random, and fractal series convert into regular, random and scale-free networks, respectively.

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The discussions mainly focus on stationary time series generated with theoretical models. When we try to deal with real world data, we should answer some important questions, such as what are the non-stationary (trend) effects on the structural properties of networks, and what can the structural properties of networks tell us about time series.

In the present paper, the visibility graph is used to analyze several exchange rate series. We find that the series convert into hierarchically structured scale-free networks. The relations between the scaling exponents of the degree distributions and the Hurst exponents obey the analytical prediction for fractal Brownian motions. The exchange rate of EURO to Japanese Yen is widely used to evaluate risk and to estimate trends in speculative investments. Interestingly, we find that the hierarchies of the series for these two currencies are significantly weak compared with that of the other series.

2. Methods and materials

Visibility graph. To keep the description as self-contained as possible, we review briefly the concept of visibility graph [12]. Let us consider a scalar time series, denoted with $\{y_i \,|i = 1, 2, \ldots, N\}$, where N is the total number of records. One can map the time series points to nodes, and link each pair of nodes if there exists visibility between them. The visibility between two arbitrary points, y_a and y_b, refers to that all the points between them fulfills,

$$y_c \leq y_b + (y_a - y_b) \cdot \frac{b-c}{b-a}. \tag{1}$$

That is, all the intermediate points do not intersect with the straight line between the two points.

Degree distribution. The constructed networks are measured by using the degree distribution [7]. Degree of a node is the number of the nodes directly connected with it. Scale-free is a characteristic shared by a large amount of real world networks. In a scale-free network, the degree distribution obeys a right-skewed power-law, $p(k) \sim k^{-\alpha}$. Consequently, there exist some high-degree nodes acting as hubs, and we cannot find a characteristic degree of the degree-distribution. Exponential distribution is another important type widely existing in the real world, namely, $p(k) \sim \exp(-\lambda k)$.

Lacasa et al. [13] present an analytical prediction of behaviors of the tail of $p(k)$. According to Eq. (1), the nodes corresponding to extreme values in the considered series have typically large degrees, and consequently dominate the behaviors of the tail. For a node corresponding to the extreme value y_e, the probability of this node to have degree k is,

$$p_{pre}(k) \sim P_{fr}(k) \cdot r(k), \tag{2}$$

where $P_{fr}(k)$ is the occurring probability of $y_{e+k} = y_e$, namely, the first return time distribution. And $r(k)$ is the percentage of visible nodes between e and $e + k$, which can be estimated with the normalized standard deviation, namely,

$$r(k) \sim \frac{1}{k} \cdot \frac{1}{N-k+1} \cdot \sum_{e=1}^{N-k+1} \left[\sum_{j=e}^{e+k-1} \left(y_j - \langle y_j \rangle\right)^2 /k \right]^{\frac{1}{2}}, \tag{3}$$

where $\langle y_j \rangle$ is the mean value of the segment, $\{y_e, y_{e+1}, \ldots, y_{e-k+1}\}$. We record the index number s if the line connecting the s'th and the $(s + 1)$'th nodes intersects with the line, $y = y_e$ and calculate the interval between each pair of successive recorded index numbers. Repeat this procedure for the values of $y_e = y_{\min} + \frac{y_{\max} - y_{\min}}{W} \cdot w$, $(w = 1, 2, \ldots, W)$. $y_{\max}$ and $y_{\min}$ are the maximum and minimum values in the series, respectively. When the repeat times W is large enough, the function $P_{fr}(k)$ can be approximated with the histogram of the calculated intervals. W is chosen to be 500 in the calculations in the present paper.

For a fractal Brownian motion (FBM) with Hurst exponent H, the predicted degree distribution obeys a power-law, namely, $p_{pre}(k) \sim k^{-\beta} \sim k^{2H-3}$. β is the predicted scaling exponent. Accordingly, we can estimate the value of the Hurst exponent, $H = \frac{3-\beta}{2}$. Because of the intrinsic non-stationary, long-range dependence and finite length of real world FBM series, characterizing these series via Hurst exponent requires sophisticated techniques that often yield ambiguous results (an overview can be found in Ref. [13] and references therein). Visibility graph proves to be a reliable method to estimate the Hurst exponents of FBMs.

Hierarchy. In a network, the nodes may cluster into some groups, and the nodes in each group are densely connected while different groups are connected loosely with few edges. Each group may also contain some small sub-groups. Occurrence at different scales of this kind of modular structures leads to hierarchy of networks [15,16]. Generally, low-degree nodes form the densely connected sub-groups, which are connected with each other by high-degree nodes. Clustering coefficient of a node is defined to be the ratio between the numbers of existing edges and possible edges among its neighbors. In hierarchical networks, clustering coefficient tends to decrease with the increase of degree. It is found in many real world networks [15–17] that the relation between clustering coefficient and degree behaves power-law, namely, $C^*(k) \sim k^{-\gamma}$. The exponent γ is widely used as a criterion to detect hierarchical structures.

Multifractal. To understand the structural characteristics of the visibility graphs, the wavelet transform (WT) [18,19] is used to detect the scaling properties of the series. The WT of the series $\{y_i \,|i = 1, 2, \ldots, N\}$ can be calculated as $W(s, a) = \frac{1}{a} \sum_{i=1}^{N-1} y_i \cdot g\left(\frac{i-s}{a}\right)$. g is the wavelet and a the given scale. We denote the maximum positions of WT with $\{s_1, s_2, \ldots, s_M\}$. For a multifractal series, in the long scale limit the partition function obeys a power-law, i.e.,

$$Z(a, q) = \sum_{s=s_1}^{s_M} |W(s, a)|^q \propto a^{\tau(q)}. \tag{4}$$