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Axiomatic approach to the cosmological constant

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ABSTRACT

A theory of the cosmological constant Λ is currently out of reach. Still, one can start from a set of axioms that describe the most desirable properties a cosmological constant should have. This can be seen in certain analogy to the Khinchin axioms in information theory, which fix the most desirable properties an information measure should have and that ultimately lead to the Shannon entropy as the fundamental information measure on which statistical mechanics is based. Here we formulate a set of axioms for the cosmological constant in close analogy to the Khinchin axioms, formally replacing the dependence of the information measure on probabilities of events by a dependence of the cosmological constant on the fundamental constants of nature. Evaluating this set of axioms one finally arrives at a formula for the cosmological constant given by $\Lambda = \frac{1}{\hbar^4} G^2 \left(\frac{m_e}{\alpha_{el}} \right)^6$, where G is the gravitational constant, m_e the electron mass, and α_{el} the low-energy limit of the fine structure constant. This formula is in perfect agreement with current WMAP data. Our approach gives physical meaning to the Eddington–Dirac large-number hypothesis and suggests that the observed value of the cosmological constant is not at all unnatural.

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1. Introduction

The cosmological constant problem is probably one of the most fundamental problems in physics that so far has resisted any attempt of solution [1]. When looking through the large amount of literature on the cosmological constant Λ and the associated cosmological constant problem, one statement is found quite regularly: The observed value of the cosmological constant (or dark energy density) that is suggested by WMAP and other astronomical observations [2,3] is regarded by most physicists to be rather unnatural and surprising, and some people from the anthropic school even regard it to be unexplainable. From a quantum field theory point of view one would have *a priori* expected a value of vacuum energy density $\rho_{\text{vac}} = \frac{c^4}{8\pi G}\Lambda$ given by something of the order m_{pl}^4 (in units where $\hbar = c = 1$), since the Planck mass m_{pl} is a suitable cutoff scale for vacuum fluctuations where quantum field theory needs to be replaced by something else. So for a quantum field theorist the observed value of the cosmological constant is surprisingly small. Astrophysicists, on the other hand, are facing a rather large value of Λ in the ΛCDM model as compared to observable mass densities. This means that the current universe is dominated by vacuum energy, whereas *a priori* most astrophysicists would have probably expected dark energy to play a less pronounced role, so for them Λ is surprisingly large. In supersymmetric theories, and in particular superstring theory, the most natural value of the cosmological constant is zero, and again it is not clear how to obtain a small positive value at the current time by a 'natural' mechanism. This has led to models based on anthropic considerations, which give up the idea of a single universe and regard Λ as a random variable whose value (by construction of the anthropic argument) can never be explained: only probabilistic statements can be given for an ensemble of 'multiverses' [4].

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Given all these controversies and mysteries surrounding the cosmological constant, it is perhaps worth going back to the basics and asking ourselves how natural or unnatural the observed value really is. Since the development of a full theory of the cosmological constant is currently out of reach, we will start by formulating a set of axioms that describe the most desirable properties a cosmological constant should have. This can be seen in analogy to the set of Khinchin axioms [5,6] in information theory that describe the most desirable properties an information measure should have. It is well known that from the Khinchin axioms one can uniquely derive the Shannon entropy, on which the entire mechanism of statistical mechanics is founded (see any textbook on the subject, e.g. Ref. [7]). Similarly, we will formulate suitable axioms for a cosmological constant. The principal idea underlying this approach is that ultimately the cosmological constant is expected to be part of a unified theory of quantum gravity and the standard model of electroweak and strong interactions. It will thus potentially depend on fundamental constants of nature. The axioms that we formulate deal with possible dependences on these fundamental parameters.

Roughly speaking, the physical contents of these axioms is as follows: The cosmological constant should only depend on fundamental parameters of nature (rather than irrelevant parameters), it should be bounded from below, it should depend on the relevant fundamental constants in the simplest possible way, and the dependence should be such that one obtains scale invariance of the universe under suitable transformations of the fundamental parameters that leave the physics invariant. We will point out that these four axioms for the cosmological constant are very similar in style to the four Khinchin axioms that ultimately underlie the foundations of statistical mechanics. Amazingly, out of the four axioms a formula for Λ can be derived that is in excellent agreement with current observations. This formula is given by

$$\Lambda = \frac{1}{\hbar^4} G^2 \left(\frac{m_e}{\alpha_{\text{el}}} \right)^6 , \tag{1}$$

where G is the gravitational constant, m_e the electron mass, and α_{el} the low-energy limit of the fine structure constant.

It turns out that the result (1) of our derivation can be interpreted as a particular form of the Eddington–Dirac large-number hypothesis, connecting cosmological parameters and fundamental constants [8–12], in a form previously advocated by Nottale [10,11], based on previous work by Zeldovich [13]. Still, our derivation is very different from Nottale's original approach, since we do not need any assumption on 'scale relativity' [11]. Rather, our method is much more related to an information-theoretic approach (similarly as in statistical mechanics) and gives new physical meaning to this kind of approach. The above value of the cosmological constant is singled out as a kind of optimum value that is consistent with the axioms. The validity of formula (1) has also been independently conjectured in a recent paper by Boehmer and Harko [14].

The vacuum energy density (dark energy density with equation of state $w = -1$) that follows from our axiomatic approach is given by

$$\rho_{\text{vac}} = \frac{c^4}{8\pi G} \Lambda = \frac{1}{8\pi} \frac{c^4}{\hbar^4} G \left(\frac{m_e}{\alpha_{\text{el}}} \right)^6 . \tag{2}$$

According to the four axioms that we will formulate in the following sections, Eq. (2) yields a kind of an optimum value of the vacuum energy in the universe, according to criteria set out by the axioms. Numerically, this formula yields the prediction

$$\rho_{\text{vac}} = (4.0961 \pm 0.0006)\ \text{GeV/m}^3. \tag{3}$$

The current astronomical measurements provide evidence for a dark energy density of about

$$\rho_{\text{dark}} = (3.9 \pm 0.4)\ \text{GeV/m}^3. \tag{4}$$

We thus conclude that the observed value of the cosmological constant is not at all unnatural, but derivable from a set of suitable axioms that make physical sense.

This paper is organized as follows. In Section 2 we briefly recall the Khinchin axioms, in order to make this paper self-contained for readers that do not have an information theory background. In Section 3 we formulate our four axioms for the cosmological constant and point out the analogy with the Khinchin axioms. In Section 4 we derive the above formula for ρ_{vac} from the axioms. In Section 5 we point out that our axiomatic approach gives physical meaning to the Eddington–Dirac large-number conjecture. Our concluding remarks are given in Section 6.

Throughout this paper our notion of ρ_{vac} means the observable (physically relevant) vacuum energy density, which should be distinguished from any bare (unmeasurable) contributions; see e.g. Ref. [15] for a discussion of this subtlety.

2. The Khinchin axioms

Khinchin [5] formulated four axioms that describe the most desirable properties an information measure I should have. These four axioms uniquely fix the functional form of the Shannon information and are extremely important for the mathematical foundations of statistical mechanics. Let us recall these four axioms as well as their physical meaning. Later, we will formulate analogous axioms for the cosmological constant.

A1: Fundamentality

$$I = I(p_1, \ldots, p_W). \tag{5}$$

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