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Transition from Pareto to Boltzmann–Gibbs behavior in a deterministic economic model

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ABSTRACT

The one-dimensional deterministic economic model recently studied by González-Estévez et al. [J. González-Estévez, M.G. Cosenza, R. López-Ruiz, J.R. Sanchez, Physica A 387 (2008) 4637] is considered on a two-dimensional square lattice with periodic boundary conditions. In this model, the evolution of each agent is described by a map coupled with its nearest neighbors. The map has two factors: a linear term that accounts for the agent's own tendency to grow and an exponential term that saturates this growth through the control effect of the environment. The regions in the parameter space where the system displays Pareto and Boltzmann–Gibbs statistics are calculated for the cases of the von Neumann and the Moore neighborhood. It is found that, even when the parameters in the system are kept fixed, a transition from Pareto to Boltzmann–Gibbs behavior can occur when the number of neighbors of each agent increases.

1. Introduction

In the last few years, different probabilistic models [1–5] have been proposed to explain wealth distribution in western societies [6–10], namely the Boltzmann–Gibbs distribution grouping about 95% of individuals, corresponding to those belonging to the low and middle economic classes, and the Pareto distribution consisting of 5% of individuals possessing the highest wealth. The majority of these models explain wealth distribution as a consequence of random processes. For example, coexistence of Gamma and Pareto distributions has been found in a probabilistic kinetic exchange model [11]. Other works assume that economic agents often make irrational, although not random choices, rather based on physiological factors [12]. However, most classical economic theories assume that economic transactions are carried out under the rationale force of some interest or some final profit, and only occasionally this is conducted by chance. Thus, in order to shed light on the problem of how wealth is distributed in human society, it is important to dispose of other economic models incorporating different degrees of determinism in the interaction among the agents.

One of these models was recently proposed [13] and studied in detail for the one-dimensional case in Ref. [14]. This is a completely deterministic model that reproduces realistic wealth distributions, i.e., the Pareto and the Boltzmann–Gibbs (BG) distributions, for different values of parameters. Moreover, it is possible to produce a transition from Pareto to BG behavior by only modifying one parameter. This means that it is possible to bring the system from a particular wealth distribution to another one with a lower inequality by making only a small change in the system configuration. This is an advantage with respect to other random models where it is necessary to perform a structural reconfiguration of the system in order to get this type of transition [2,3].

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In this paper, we ask whether some other strategies can be implemented in this model [13,14] in order to induce a transition in its asymptotic statistical behavior. We proceed to affirmatively answer this question by showing that different local groupings of the agents that occur, for instance, by changing the topology of the lattice or the number of neighbors of each agent, can lead to a transition from the Pareto to BG behavior even when the parameters of the system are kept fixed. Hence, the first step we undertake in Section 2 is the characterization of the statistical behaviors that this model exhibits on its space of parameters when it is implemented on a two-dimensional lattice. Particularly, the cases of the von Neumann and the Moore neighborhood are studied in detail. Then, in Section 3, the results obtained for these configurations are compared with those previously found for the one-dimensional case. The regions in the parameter space where a transition between different statistical behaviors can take place, are identified in this section. Section 4 contains our conclusions.

2. The deterministic economic model on a two-dimensional lattice

The system [13,14] consists of N agents placed at the nodes of a network. Here, the network is a square lattice with periodic boundary conditions. Each agent, representing an individual, a company, a country or other economic entity, is identified by a pair of indices (i, j), with $i, j = 1, \dots, N$. The dynamics of each agent is described by a discrete-time map that expresses the competition between its own tendency to grow and an environmental influence that controls this growth. The dynamics of the system is described by the coupled map equations

$$\begin{aligned} x_{t+1}^{i,j} &= r_{i,j} x_t^{i,j} \exp(-|x_t^{i,j} - a_{i,j} \Psi_t^{i,j}|), \\ \Psi_t^{i,j} &= \frac{1}{\eta(i,j)} \sum_{i,j \in \nu(i,j)} x_t^{i,j}, \end{aligned} \tag{1}$$

where $x_t^{i,j} \geq 0$ gives the state of the agent (i, j) at discrete time t, and it may denote the *wealth* of this agent; the factor $r_{i,j} x_t^{i,j}$ expresses the *self-growth capacity* of agent (i, j), characterized by a parameter $r_{i,j}$; $\Psi_t^{i,j}$ represents the local field acting at the site (i, j) at time t; $\nu(i, j)$ is the set of agents in the network coupled to agent (i, j) and $\eta(i, j)$ is the cardinality of this set; and $a_{i,j}$ measures the coupling of agent (i, j) with its neighborhood; it can also be interpreted as the *local environmental pressure* exerted on agent (i, j) [15]. The negative exponential function acts as a *control factor* that limits this growth with respect to the local field. With the dynamics given by Eqs. (1) the largest possibility of growth for agent (i, j) is obtained when $x_t^{i,j} \simeq a_{i,j} \Psi_t^{i,j}$, i.e., when the agent has reached some kind of adaptation to its local environment. Thus our model implicitly links individual growth with social prosperity, a dilemma commonly found in models of evolutionary game theory [16].

We consider two different nearest-neighbor interactions: the 4-cell von Neumann neighborhood,

$$\Psi_t^{i,j} = \frac{1}{4} \left(x_t^{i-1,j} + x_t^{i+1,j} + x_t^{i,j-1} + x_t^{i,j+1} \right); \tag{2}$$

and the 8-cell Moore neighborhood,

$$\Psi_t^{i,j} = \frac{1}{8} (x_t^{i-1,j} + x_t^{i+1,j} + x_t^{i,j-1} + x_t^{i,j+1} + x_t^{i-1,j-1} + x_t^{i-1,j+1} + x_t^{i+1,j-1} + x_t^{i+1,j+1}). \tag{3}$$

We focus on a homogeneous system where all agents possess the same growth capacity, $r_{i,j} = r$, and are subject to a uniform selection pressure from their environment, $a_{i,j} = a$. Thus, the parameter a can be interpreted as a homogeneous social constraint on the agents to reach a wealth level proportional to that of their environment. The value $a = 1$ implies a tendency towards being totally balanced with the neighborhood. The case $a < 1$ corresponds to a lower level of expectation among agents that restricts the possibilities for improving their relative wealth. When $a > 1$ the agents possess a greater stimulus for overcoming their local neighbors.

We study the collective behavior of the system described by Eqs. (1) in the space of parameters (a, r). For all the simulations shown, the system size is 316×316 ($N \simeq 10^4$) and the values of the initial conditions are uniformly distributed at random in the interval $x_0^{i,j} \in [1, 100]$. Also, a transient of 10^4 iterations is discarded before arriving at the asymptotic regime where all the calculations are carried out. When indicated, time averages are done over the next 100 iterations after the transient, and this result is newly averaged over 100 different realizations of the initial conditions with the same parameter values.

Fig. 1 shows the regions where the probability distribution $P(x)$ displays BG behavior, $P(x) \sim e^{-\mu x}$, and Pareto behavior, $P(x) \sim x^{-\alpha}$, in the space of parameters (a, r) for the system equations (1). The parameters a and r are varied in intervals of size $\Delta a = 0.02$ and $\Delta r = 1$, respectively. For each pair (a, r), and after undergoing the averaging process described above, semilog and log–log linear regressions are used to calculate the exponents μ and α for the corresponding distributions, and only those results yielding a correlation coefficient greater than 0.96 are shown in Fig. 1.

In comparison with the one-dimensional case [14], the BG and Pareto regions in the space of parameters are larger for the square lattice. In both cases, the BG behavior occurs for the lower values of the parameter a, but in two dimensions there is also a narrow BG region for greater values of a. In general, when the value of a increases, the population enters a competitive regime that provokes the appearance of the Pareto behavior. The scaling exponents obtained for both regions are similar to the one-dimensional case. Those exponents that correspond to the Pareto behavior are in the range $\alpha \in [2.4, 3.0]$, in agreement with those found in actual economic data [7,17,18].