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The study of dynamic singularities of seismic signals by the generalized Langevin equation

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ABSTRACT

Analytically and quantitatively we reveal that the generalized Langevin equation (GLE), based on a memory function approach, in which memory functions and information measures of statistical memory play a fundamental role in determining the thin details of the stochastic behavior of seismic systems, naturally leads to a description of seismic phenomena in terms of strong and weak memory. Due to a discreteness of seismic signals we use a finite–discrete form of the GLE. Here we studied some cases of seismic activities of Earth ground motion in Turkey with consideration of the complexity, nonergodicity and fractality of seismic signals.

1. Introduction

Specific stochastic dynamics occur in a large variety of systems, such as supercooled liquids, seismic systems, the human brain, finance, meteorology and granular matter. These systems are characterized by an extremely rapid increase or a slowdown of relaxation times and by a non-exponential decay of time-dependent correlation functions [1,2].

The canonical theoretical framework for stochastic dynamics of complex systems is the time-dependent generalized Langevin equation (GLE) [3–7,14,15]. It successfully describes the phenomenon of statistical memory, whereby the relaxation time for order parameter fluctuations scales as a power of the correlation length. An obvious question to ask would be whether this framework can be adapted to describe seismic phenomena. Analytically and quantitatively we show that the GLE, based on a memory function approach, where the memory functions and information measures of statistical memory play a fundamental role in determining the thin details of the stochastic behavior of seismic systems, naturally leads to a description of seismic phenomena in a terms of a strong and weak memory. Due the discreteness of a seismic signals we use a finite–discrete form of the GLE. Here we study some cases of seismic activities of Earth ground motion in recent years in Turkey with consideration of the complexity, irregularity and metastability of seismic signals.

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2. Some extraction from the theory of discrete stochastic processes

The GLE analytical model was originally proposed for displaying the stochastic behavior of signals in complex systems [3–7].

Here we consider the data of seismic signal recording as a time series ξ:

$$\xi = \{\xi_0, \xi_1, \xi_2, \dots, \xi_{N-1}\} = \{\xi(0), \xi(\tau), \xi(2\tau), \dots, \xi([N-1]\tau)\}. \tag{1}$$

Here τ is the discretization time of seismic signals, and N is the total number of signals. The set of fluctuations $\delta\xi$ is an initial dynamic variable W_0:

$$W_0 = \{\delta\xi_0, \delta\xi_1, \delta\xi_2, \dots, \delta\xi_{N-1}\}, \quad \delta\xi_j = \xi_j - \langle\xi\rangle, \quad \langle\xi\rangle = \frac{1}{N}\sum_{j=0}^{N-1}\xi_j. \tag{2}$$

The Gram–Schmidt orthogonalization procedure

$$\langle W_n, W_m\rangle = \delta_{n,m}\langle|W_n|^2\rangle \tag{3}$$

leads to the set of the following orthogonal dynamic variables:

$$\begin{cases} W_0 = \delta\xi, \\ W_1 = \mathcal{L}W_0 = \dfrac{\mathrm{d}}{\mathrm{d}t}\delta\xi, \\ W_2 = \mathcal{L}W_1 - \Lambda_1 W_0, \\ \dots, \\ W_{n+1} = \mathcal{L}W_n - \Lambda_n W_{n-1}, \quad n \geq 1, \end{cases} \tag{4}$$

where $\mathcal{L} = (\Delta - 1)/\tau$ is the Liouville quasioperator and Λ_n is the relaxation parameter of the nth order (where Δ is the shift operator $\Delta x_j = x_{j+1}$ and τ is the discretization time).

Within the framework of statistical theory and Zwanzig–Mori's theoretical–functional procedure of projection operators, one can obtain the following recurrent relation as a finite-difference kinetic equation:

$$\Delta M_n(t) = \tau\lambda_{n+1}M_n(t) - \tau^2\Lambda_{n+1}\sum_{j=0}^{m-1}M_{n+1}(t - j\tau)M_n(j\tau), \quad n = 0, 1, 2, \dots. \tag{5}$$

Here we introduce a Liouville quasioperator eigenvalue λ_{n+1}, a relaxation parameter Λ_{n+1} and a memory function $M_n(t)$ of the nth order, respectively:

$$\lambda_n = \frac{\langle W_{n-1}\mathcal{L}W_{n-1}\rangle}{\langle|W_{n-1}|^2\rangle}, \qquad \Lambda_n = \frac{\langle|W_n|^2\rangle}{\langle|W_{n-1}|^2\rangle}, \qquad M_n(t) = \frac{\langle W_n(t)W_n\rangle}{\langle|W_n|^2\rangle}. \tag{6}$$

For analysis of the relaxation time scales of the underlying processes we use the frequency-dependent statistical non-Markovity parameter $\varepsilon_n(\omega)$:

$$\varepsilon_n(\omega) = \left\{\frac{\mu_{n-1}(\omega)}{\mu_n(\omega)}\right\}^{1/2}. \tag{7}$$

Here $\mu_n(\omega)$ is a frequency power spectrum for the memory function of the nth order:

$$\mu_n(\omega) = \left|\tau\sum_{j=0}^{N-1}M_n(j\tau)\cos(j\tau\omega)\right|^2. \tag{8}$$

Using Eqs. (1)–(8) we can study all specific singularities of the statistical memory effects in an underlying system. The non-Markovity parameter and its statistical spectrum were introduced by Yulmetyev et al. in Ref. [8]. It is worth mentioning that the non-Markovian character of seismic data was discussed by Varotsos et al. [9]. One of the first proofs of non-Markovity of empirical random processes was given in Refs. [10]. Stochastic origins of the long-range correlations of ionic current fluctuations in membrane channels with non-Markovian behavior were studied in Ref. [11].

3. An analysis of results

Fig. 1 presents the initial time series of seismic signals for seven seismic origins: *grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, and *gdz*. The discretization time is $\tau = 0.02$ s. We can see that all the time series have distinctive features.

Fig. 2 demonstrates the frequency dependence of the first point of the non-Markovity parameter $\varepsilon_1(\omega)$ for seven seismic origins from Turkey: *grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, and *gdz*. Since the nature of each seismic source is unknown to us, it would be interesting to establish its character. It seems possible that the signals can be distributed into three groups: **group A** (*kelt*, *gdz*), **group B** (*grsn*, *sgkt*, *uldt*, *seyt*), and **group C** (*mack*).