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Invasion percolation between two sites in two, three, and four dimensions

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ABSTRACT

The mass distribution of invaded clusters in non-trapping invasion percolation between an injection site and an extraction site has been studied, in two, three, and four dimensions. This study is an extension of the recent study focused on two dimensions by Araújo et al. [A.D. Araújo, T.F. Vasconcelos, A.A. Moreira, L.S. Lucena, J.S. Andrade Jr., Phys. Rev. E 72 (2005) 041404] with respect to higher dimensions. The mass distribution exhibits a power-law behavior, $P(m) \propto m^{-\alpha}$. It has been found that the index α for $p_e < p_c$, p_c being the percolation threshold of a regular percolation, appears to be independent of the value of p_e and is also independent of the lattice dimensionality. When $p_e = p_c$, α appears to depend marginally on the lattice dimensionality, and the relation $\alpha = \tau - 1$, τ being the exponent associated with cluster size distribution of a regular percolation via $n_s \propto s^{-\tau}$, appears to be valid.

1. Introduction

The invasion percolation model was introduced by Wilkinson and Willemsen [1] in order to describe more realistically in microscopic scale the displacement of fluid–fluid interfaces with respect to the multiphase flow in porous media than regarding a regular percolation model [2,3]. Examples regarding multiphase flow are the water flow through cracked rocks and underground oil and gas flow through underground pores. Information on multiphase flow is of great importance in exploring underground resources, such as petroleum or natural gas, and the exploration of such resources can be efficiently performed by injecting water or immiscible gas (*e.g.*, carbon dioxide or methane) into an injection well until the injecting fluid penetrates pores and reaches an extraction well. When a wetting (invading) fluid, such as water, is injected through porous medium saturated with a non-wetting (defending) fluid, such as oil, the viscous force is dominated by the capillary force acting on the interface of two fluids. Thus, the capillary force acts as a driving force. This situation can be modeled by a lattice network, in which each lattice site is initially filled with a defending fluid and the invading fluid invades the least pressurized (or the largest pore) sites neighboring the invaded sites.

In a regular lattice percolation scenario, each lattice site (bond) is either occupied with a probability p or unoccupied with a probability $1 - p$. The two neighboring occupied sites (bonds) are assumed to be connected, and belong to the same cluster; thus, tuning the occupation probability p, the system undergoes a phase transition from an unpercolating state to a percolating state at the percolation threshold p_c. The universality class is classified by the static critical exponents ν, β, and γ, associated, respectively, with correlation length, order parameter, and susceptibility, near p_c. The spanning length of the largest cluster, the probability of an arbitrary site belonging to an infinite cluster, and the second moment of the cluster size distribution are considered to be the correlation length, order parameter, and susceptibility, respectively. These exponents are related to the fractal dimension of the infinite cluster via $d_f = d - \beta/\nu$, with d as the underlying lattice dimensionality.

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On the other hand, in an invasion percolation, each nearest-neighbor site with the least amount of pressure is invaded, and the process continues until the stopping criterion is achieved; thus, no tuning parameter is necessary and the system evolves into the natural critical state, i.e., into the self-organized criticality [4]. The invasion process begins from a single site and all invaded sites are connected, forming a single cluster; therefore, the order parameter and the susceptibility are not defined. The universality of an invasion percolation has been thus clarified by the quantities differing from those of a regular percolation. The fractal dimensions of a connected network, a backbone, and an elastic backbone were instead measured [1,5–7]. Various critical indices associated with various characteristic paths, such as the optimal path and the shortest path between two sites, were also measured in order to clarify the universality class [8,9].

Two variants of invasion percolation, i.e., the trapping invasion percolation (TIP) and the non-trapping invasion percolation (NTIP), have been studied. The former is relevant to the situation in that both the defending fluid and the invading fluid are incompressible, while the latter is relevant to the situation in that the defending fluid is infinitely compressible and the invading fluid is incompressible. In regards to the former case, when the defending fluid is surrounded in a pore by the invading fluid, the invading fluid cannot invade the pore, whereas in regards to the latter case the invading fluid can invade all pores, whether or not they are filled with defending fluid. These two models were known to display different critical behaviors, such as that the fractal dimensions of invaded clusters were known to be different from each other. While the fractal dimension of the TIP clusters was $d_f = 1.82$ on a two-dimensional substrate [5,6], that of NTIP cluster was found to be close to the fractal dimension of an infinite network of a regular percolation at criticality [1,2,9]. Contrary to this, it was also reported that the scaling properties of the TIP cluster were non-universal, and its fractal dimension crossed over to the value of a regular percolation for large coordination numbers [10].

Recently, an invasion percolation model between an injection site and an extraction well, as well as that with multiple wells were investigated in two dimensions [11,12]. It was found that the fractal dimension of an invaded cluster was similar to the fractal dimension of an infinite cluster of a regular percolation, regardless of the pressure at the extraction site. On the other hand, the distribution of masses of an invading fluid was found to exhibit dramatically different behaviors depending on the pressure of the extraction site. For the pressure of the extraction site $p_e = 0$, it exhibited the power-law behavior $P(m) \propto m^{-\alpha}$ with the power $\alpha \simeq 1.39$, whereas for $p_e = p_c$, p_c being the regular percolation threshold, it still exhibited the power-law behavior but with a different power, $\alpha \simeq 1.01$ [11]. The power α for the latter case was conjectured to be related to the exponent τ, describing the cluster size distribution of a regular percolation as $n_s \sim s^{-\tau}$, via $\alpha = \tau - 1$, with n_s being the number of clusters of size s per site.

In this paper, we studied the NTIP between two sites in regards to two, three, and four dimensions, focusing on ways in which the power-law behavior regarding the distribution of masses of invaded clusters varies as the lattice dimensionality increases. Our work is an extension of the recent study in two dimensions by Araújo et al. [11] to higher dimensions. We measured the mass distribution of invaded clusters. It was found that the exponent α appears to be ≈ 1.41 irrespective of the lattice dimensionality for the case of $p_e = 0$, whereas it varies, depending on the lattice dimensionality for $p_e = p_c$.

2. Model and methods

With regard to the d dimensions, the hypercubic lattice of L^d sites was used as a pore site network, which was assumed to be filled with defending fluids. An injection site and an extraction site, separated by a distance r, were set at the center of the lattice. Since the previous research [11] determined that the power-law region becomes narrower as the distance r increases, leaving the power of the distribution of masses unaffected, we choose only two values of r, $r = 2$ and $r = 4$, throughout the work. The random numbers between 0 and 1, representing the pressures (or the pore site sizes), are distributed on the lattice sites of the system. Beginning from the injection site, the neighboring sites are listed in descending order, from the highest pressure site to the lowest pressure site. In regards to the next and forthcoming steps, the lowest-pressure site is invaded and the new sites neighboring the one just invaded are added in the list, therefore, maintaining the order from the highest pressure to the lowest pressure. This process is continued until the extraction site is invaded, at which time the process is terminated and the mass of invaded clusters is sampled.

Since the invasion process is begun from the center of the lattice, it might be possible that the invaded clusters quickly reach the boundary of the finite system before the extraction site is invaded. Whenever this happened, the process was stopped and the sample was discarded in the earlier studies [11]. This prescription apparently yields a sharp drop in the large mass region of the distribution due to a lack of data for large clusters, thereby severely reducing the power-law region. This type of bias is even more significant in regards to higher dimensions because the linear system size will be smaller, due to computer memory and computing time limitations. In order to reduce such a finite-size effect, we employed periodic boundary conditions in this study. When an invaded cluster reaches the boundary and the neighboring site out of the given cell is about to be invaded, the image site on an opposite edge is invaded unless the invaded network wraps the system. This prescription allows us to sample the data of large masses, but requires a relatively long computing time. In order to avoid wasting the sampling time for rare events for which almost an entire lattice may be invaded, a cutoff mass is set. When the mass of invaded network reaches the cutoff mass, the process is stopped and the sample is discarded. This prescription does not affect the power-law behavior but only limits the power-law region up to the cutoff mass.