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Stochastic resonance in a bias monostable system with frequency mixing force and multiplicative and additive noise

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ABSTRACT

The stochastic resonance in a bias monostable system subject to frequency mixing force and multiplicative and additive noise is investigated. Based on the adiabatic elimination theory, the analytic expressions of the signal-to-noise ratio (SNR) for the fundamental and higher harmonics are obtained. It is shown that the SNR is a non-monotonic function of the intensities of the multiplicative and additive noise, as well as the system parameter. Moreover, the SNR for the fundamental harmonic decreases with the increase of the system bias, while the SNR for the higher harmonics behaves non-monotonically as the system bias varies.

1. Introduction

Stochastic resonance (SR) is a phenomenon arising due to the entanglement between noise and non-linearity of a system, in which the strength of a suitably defined output signal is maximum for optimum non-zero noise intensity [1]. The study of stochastic resonance in a bistable system with several periodic forces has attracted great attention [2–5]. Landa and McClitock [2] found the vibrational resonance in an over-damped bistable system only subject to two periodic fields. Gitterman [3] developed the theoretical results of a bistable oscillator driven by two periodic forces. Grigorenko et al. [4,5] investigated the response of a bistable system with a frequency mixing force. Strier et al. [6] presented an analytical study of the enhancement of the signal-to-noise ratio (SNR) in a monostable non-harmonic potential. They made use of the exact expression for the diffusion propagator obtained in a previous work, and found a monotonically increasing response with the noise amplitude. For the first time, they provided a cut-off to such an increase, which prevents a probability leakage out of the system. Conventional SR is a non-linear effect that accounts for the optimum response of a dynamical system to an external force at certain noise intensity. The SR in a broad sense means the non-monotonic behavior of the output signal as a function of some characteristics of the noise (noise intensity or noise correlation time) or of a periodic force (amplitude or frequency).

In actual systems there are a lot of monostable systems [7–15], including chemical, electronic, physical and biological systems. Dykman et al. [7] and Evstigneev et al. [9] investigated the SR in a monostable over-damped system based on linear response theory. Stocks et al. investigated the zero-dispersion stochastic resonance (ZDSR) in a monostable system [14,15], for which the dependence of eigenfrequency upon energy has an extremum. It is well known that the multiplicative noise often plays a different role on the output of a system, with respect to the additive noise. Therefore, the investigation of the response of a monostable system driven by multiplicative noise is of great significance. In this paper, based on the adiabatic

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approximation theory, we study the SR in a bias monostable system driven by two periodic forces as well as multiplicative and additive white noise.

2. The monostable system and its signal-to-noise ratio

Consider an over-damped monostable system [8] with multiplicative and additive noise described by the following Langevin equation:

$$\dot{x} = -ax^3 + x\xi(t) + \eta(t) + f(t) + b,$$
$$f(t) = A_1\cos(\Omega_1 t) + A_2\cos(\Omega_2 t), \tag{1}$$

where $a > 0$ is a system parameter, and b is a constant force, denoting the bias of the monostable system. The noise terms $\xi(t)$ and $\eta(t)$ are uncorrelated noise with zero mean and they are characterized by their variance

$$\left\langle \begin{bmatrix} \xi(t_1) \\ \eta(t_1) \end{bmatrix} \begin{bmatrix} \xi(t_2) & \eta(t_2) \end{bmatrix} \right\rangle = \delta(t_1 - t_2) \begin{bmatrix} 2D & 0 \\ 0 & 2P \end{bmatrix}. \tag{2}$$

Here D and P are the intensities of the multiplicative and additive noise, respectively.

According to Eqs. (1) and (2), the corresponding Fokker–Plank equation of the monostable system, Eq. (1), can be written as

$$\frac{\partial \rho(x,t)}{\partial t} = -\frac{\partial}{\partial t}\left[F(x,t)\rho(x,t)\right] + \frac{\partial^2}{\partial x^2}\left[G(x)\rho(x,t)\right], \tag{3}$$

where

$$F(x,t) = Dx - ax^3 + f(t) + b, \qquad G(x) = Dx^2 + P. \tag{4}$$

We assume that the external force frequency is so small that there is enough time for the system to reach the local equilibrium during the period of the external force, i.e., we make the assumption that the system satisfies the adiabatic approximation condition [16]. The asymptotic long-time distribution function can be derived from Eqs. (3) and (4) in the adiabatic limit, i.e.,

$$\rho_{st}(x) = \frac{C}{[G(x)]^{1/2}} \exp\left[-\frac{V(x)}{D}\right], \tag{5}$$

where C is the normalization constant, and $V(x)$ is the rectified potential function, which has the form

$$V(x) = \int_{-\infty}^{x} \frac{D\left[-U'(x) + f(t) + b\right]}{G(x)} \mathrm{d}x, \tag{6}$$

with

$$U'(x) = \frac{\mathrm{d}U}{\mathrm{d}x} = ax^3 - Dx. \tag{7}$$

From Eqs. (6) and (7), one can see that, for the case of $D \neq 0$, i.e., in the presence of multiplicative noise, the monostable system (1) can thus be regarded as an equivalent bistable system, i.e., corresponding to the so-called two-state model [16], with $x_u = 0$ and $x_\pm = \pm\sqrt{D/a}$ being the unstable and stable states of the equivalent bistable system. Under the adiabatic limit condition, the transition rates out of $x_\pm$ can be obtained by

$$N_\pm(t) = \frac{\left|\left[U''(x_u)U''(x_\pm)\right]\right|^{1/2}}{2\pi} \exp\left[-\frac{V(x_u) - V(x_\pm)}{D}\right]$$
$$= N_{\pm 0}\exp\left[\mp k f(t)\right], \tag{8}$$

where $N_{\pm 0}$ denotes the characteristic switching frequency of the equivalent bistable system when it is only driven by multiplicative and additive noise, which is given by

$$N_{\pm 0} = \frac{D}{\sqrt{2}\pi} \exp\left[-\frac{\Delta\Phi}{2D} \mp kb\right], \tag{9}$$

with

$$k = \frac{1}{\sqrt{DP}} \arctan\left(\frac{D}{\sqrt{aP}}\right), \qquad \Delta\Phi = \left(D + \frac{aD}{P}\right)\ln\left(\frac{D^2}{aP} + 1\right) - D. \tag{10}$$