Physica A 388 (2009) 2321–2330

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

On a unified theory of cold dark matter halos based on collisionless Boltzmann–Poisson polytropes

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ARTICLE INFO

Article history:
Received 29 September 2008
Received in revised form 15 January 2009
Available online 13 March 2009

PACS:
05.20.Dd
51.10.+y
05.60.Cd
52.65.Ff
52.25.Fi
95.35.+d
98.10.+z
04.40.-b

Keywords:
Dark matter halos
Polytropes
Vlasov–Poisson
Boltzmann–Poisson

ABSTRACT

Collisionless particles in dark matter halos constitute ideal systems for applying the theory of collisionless Boltzmann–Poisson (BP) polytropes. This analysis provides a powerful complementary method for studying galactic halos. A comparison of the results obtained here and the Navarro, Frenk and White (NFW), Isothermal and Burkert profiles is shown. We obtain very good agreement with NFW profiles with errors of the order of 3%. The best polytropic profile for a finite mass halo is close to the Plummer/Schuster profile. We can explore in detail the central region and find the inner profile that complements the NFW profile. A simple formula for the inner region to which the NFW should converge for $R \to 0$ could be $\rho = \rho_0 \left(1 - (R/C)^2\right)^{4.7}$, ρ_0 and C being constants. Boltzmann–Poisson polytropes provide a theoretical approach fully compatible with the universal NFW profiles at intermediate radii and complementing them at low radii: they permit the determination of the density profile in the inner kiloparsec inaccessible to N-body simulations because of resolution limits.

1. Introduction

The aim of this paper is to give some insight into the controversy as regards N-body simulations and observational models of cold dark matter (CDM) halos, by considering polytropic gas spheres associated with the collisionless gravitational Boltzmann–Poisson (BP) system. This is a kinetic model arising as the limit of the N-body problem, also known as the Vlasov–Poisson system. The polytropic profiles have been proposed for modeling a wide variety of concrete applications in different fields; see for example Refs. [1–3]. Nevertheless, the use of polytropes is under discussion in some areas, even in the context of dark matter halos [4]. As regards this point, we will compare our results here with those of Ref. [4] at the end of the paper.

Our study confirms the results obtained by simulations over a very wide range of galactocentric radii. We provide density CDM profiles in very good agreement with both numerical (NFW) and other observational results with errors less than 3%. Then, BP polytropes provide a complementary scenario where simulations and observations might be unified. With this approach our resulting polytropic model is then used to make predictions for the behaviour of the CDM halos in those regions in which the *N-body simulation* models cannot produce detailed results, i.e. near the center, $\leq$ 1 kpc (due to

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doi:10.1016/j.physa.2009.02.045

resolution limitations), and at the rim (as halos cannot have infinite extent). This provides complementary information where other models present difficulties for making predictions. Our opinion is that with the use of polytropic models in the study of DM a complementary tool is at our disposal for considering more detailed and realistic halos by using recent and future observations. We claim that the strength of our approach lies in its flexibility. Here we have restricted ourselves to the simplest case – namely that of spherically symmetric halos – but our theoretical framework can be extended to cover many more cases, such as elliptical configurations, which are also in good agreement with recent observations [5]. Once the appropriate solution of the BP equation is chosen, the required numerical calculations are much more affordable than an N-body simulation, and we have several tools to use in discussing all relevant stability issues. The key point here is that once the shape of the halo is established, the only thing remaining is to solve an elliptical (Poisson) partial differential equation instead of a non-linear evolution (Boltzmann) equation coupled with the above Poisson equation.

It is widely accepted that CDM halos exist. This was already found in the early analytical calculations focused on the scale free nature of the gravitational collapse; see for example Refs. [6,7]. Cosmological N-body simulations (the seminal works by Navarro, Frenk and White (NFW) [8,9] being the most representative) also have confirmed that galaxies are surrounded by an extended massive dark matter halo. This is the most accepted interpretation of the flat rotation curve of spiral galaxies, even if other scenarios cannot be disregarded; see Ref. [10] for an extensive review. The existence of dark matter in the Universe has been demonstrated by the space mission WMAP observing CMB [11]. The so called NFW profiles are found to be universal, which means that they hold for a very large span of scales of dark halos, ranging from dwarf galaxies (a few kiloparsecs) to rich clusters of galaxies (several megaparsecs). See for instance the results of the "Millennium" simulation with 10 billion particles [12]. Other possibilities are, for instance, the so called "Isothermal non-singular" profile and Burkert's [13] profile. These three profiles will be addressed in this paper.

NFW profiles seem to explain very reasonably the dark matter (DM) distribution in clusters as demonstrated by weak lensing observations (e.g. Ref. [14]) and by X-ray observations (e.g. Ref. [15]). For galaxies, the agreement between simulations and observations is still under debate. The rotation curve (RC) of spiral galaxies is the main observational tool with which to look for this agreement. CDM models do not satisfactorily explain the Tully–Fisher relation [16] and even have difficulties in producing large disks [17]. Probably, the explanation of rotation curves without considering magnetic fields would be unrealistic [18,19]. On the other hand, as discussed below, the steep rise of rotation velocity in the central part has been claimed to be incompatible with the simulation outputs. Nevertheless, the DM explanation of the flat rotation curve remains the most accepted one. Reviews on rotation curves are given in Refs. [10,20].

One of the goals of this work concerns the central parts of galaxies where the resolution of numerical simulations ($\geq$1 kpc) avoids giving firm predictions and has a complex dynamics. In particular, if we define the slope in a log–log plot of density versus radius as $\gamma = -\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln R}$, the NFW profile gives $\gamma_0 = \gamma(R = 0) = 1$, while the Isothermal and Burkert profiles give $\gamma_0 = 0$. Observations also seem to suggest lower or vanishing values. Values of $\gamma_0 > 0$ correspond to "cuspy" halos, as the density becomes infinite for $R = 0$. Values of $\gamma_0 = 0$ correspond to halos with a core, the core being a region in which the density is nearly constant.

An infinite value for the density seems to be unphysical (we will discuss this later), which implies $\gamma_0 \leq 0$, and continuity arguments (in the first derivative) clearly indicate $\gamma_0 = 0$ at the very center. Near this inner region with size less than the spatial resolution of the simulations, the NFW profiles must break down and converge to a function with $\mathrm{d}\rho/\mathrm{d}R(R = 0) = 0$.

Of particular interest are the low surface brightness galaxies (LSB) as the contribution of the stellar component is so low than these galaxies are assumed to be DM dominated. In Refs. [21–23] it has been found that $\gamma_0 = 0.2 \pm 0.2$ for LSB galaxies and it was shown that even considering the influence of non-circular motions, asymmetries and offsets between optical and dynamical centers, the values close to vanishing are incompatible with NFW halos. These effects can indeed be very large. Non-circular motions are very large even in normal non-active galaxies (see e.g. Ref. [24]) as shown by bidimensional spectroscopy.

For high surface brightness, the results in Refs. [25,26] also found this incompatibility between rotation velocities and cuspy simulated halos. For in our galaxy, Ref. [27] also concluded that cuspy halos are inconsistent with observational data. It is worth recalling that the early interpretation of rotation curves in terms of DM determinations adopted the hypothesis of a maximum disc (see Ref. [28]), i.e. assumed that the contribution of DM was negligible in the center, and the results were considered as acceptable. More recently, Ref. [29] found that the dynamics of bars also favours the maximum disc hypothesis, with no need for DM in the very center. Clearly, these holes in the center are inconsistent with cusps. However, the present situation remains unclear. Ref. [30] found γ_0 between 1 (as in NFW) and zero; see Refs. [31,32]. Observational studies like that in Ref. [33] and others showed that rotation curves are not manifestly inconsistent with $\gamma_0 = 1$. Another indirect interesting approach was shown by Ref. [34], introducing constraints arising from the radial velocity dispersion; $\gamma_0 \leq 0.58$ was obtained. Therefore, there are at present large discrepancies as regards whether the central dynamics of spirals are in contradiction with cuspy halos: cusps or cores?

Numerical codes based on cold dark matter produce central cuspy density profiles of the type $\rho(r)$ proportional to r^{α}. The value of α varies from $\alpha = -1$ (for NFW profiles [8,9]) to $\alpha = -1.5$ [31,32,35]. This is in clear contrast with observations based on the rotation curve of spiral galaxies, which require a core with a constant density or slightly decreasing with r, i.e. with α close to zero. This is particularly clear in the case of dwarf and low surface brightness spiral galaxies (see for example Refs. [21,22,36–38]), but the discrepancy is also observed in normal spirals [25,26]. Cusps are also inconsistent with observations in our galaxy [27]. We refer the reader to Ref. [22] for a detailed discussion about this discrepancy. The numerically obtained cuspy profiles could be considered unacceptable from the physical point of view, as the density