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Generalized Pearson distributions for charged particles interacting with an electric and/or a magnetic field

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ABSTRACT

The linear Boltzmann equation for elastic and/or inelastic scattering is applied to derive the distribution function of a spatially homogeneous system of charged particles spreading in a host medium of two-level atoms and subjected to external electric and/or magnetic fields. We construct a Fokker–Planck approximation to the kinetic equations and derive the most general class of distributions for the given problem by discussing in detail some physically meaningful cases. The equivalence with the transport theory of electrons in a phonon background is also discussed.

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1. Introduction

It is nowadays widely accepted that the Boltzmann equation constitutes a very powerful mathematical model to study different kinetic processes like fluid-dynamic problems, chemical and nuclear reactions, diffusion and others [1]. Notwithstanding, the complicated mathematical structure embodied in this equation makes it hard to obtain explicit solutions for specific problems, both from an analytical or a numerical point of view.

In Ref. [2], a formalism was developed to introduce consistently the inelastic interactions in the Boltzmann equation. In particular, a mixture of *test particles* (TP) which spread in a medium of *field particles* (FP) endowed with two levels of internal energy was considered. In this way, under suitable hypotheses, a system describing the transport equation for the diffusion of TP in the medium has been derived.

A particular simple assumption, but still preserving physical interest, that the mass M of FP is much greater than the mass m of TP, gives rise to a model in which the TP can gain or lose a fixed amount of energy.

In this paper, we present a generalization of the results given in Ref. [2] by considering the case of charged TP diffusing in a medium (still made by FP endowed with two energy levels), whose interactions are modeled not only by means of an inelastic collision integral (as in Ref. [2]) but also by the presence of elastic collision.

In the most general fashion, we assume the presence of an external electric and/or magnetic field interacting with the system.

In the case of TP interacting with FP only by means of elastic scattering, it is usual to adopt a Fokker–Planck approximation for the collision integral. Such approximation leads to a solvable equation for the distribution function (see Refs. [3,4]). As

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it will be shown in this paper, the same approximation can be employed also in presence of the inelastic scattering, which again leads to a solvable equation for the distribution function.

Depending on the type of interactions occurring in the system, we obtain several physically relevant distributions. Among them, the generalized Pearson distribution [5,6] and the Margenau–Druyvesteyn distribution [4] are obtained in the case of hard sphere interactions, whilst power-law distributions [7,8] and modified power-law distributions [9] are derived in the case of Maxwellian interactions.

Under the above mentioned hypotheses on the masses of TP and FP, the theory we are presenting can describe the diffusion of light particles in a heavy medium, so that both a loss and a gain of a fixed amount of energy is possible. In this way, we can show how the present model is connected with the transport theory of electrons in a semiconductor lattice [10,11].

The paper is organized as follows. Section 2 is devoted to the description of the physical situation we deal with. In Section 3, by employing the truncated spherical harmonic expansion (P_1 approximation), we derive a system for the first two components $N(v)$ and $\boldsymbol{J}(v)$ of the distribution function $f(\boldsymbol{v})$. Then, by using the same method which is commonly applied to in the presence of elastic collisions only, we construct in Section 4 a Fokker–Plank approximation for the general case containing both the elastic and the inelastic terms. Explicit distributions corresponding to hard sphere and Maxwell interaction law, in the presence of electric and/or magnetic fields and elastic and/or inelastic scattering, are derived in Section 5, whilst in Section 6, we obtain the expression of the particles and heat currents related to some of these distributions. In Section 7, we study the proprieties of the inelastic collision integral only and discuss some aspects like the collision invariants and the trend to equilibrium. Finally, in Section 8, we show the mathematical equivalence of the Boltzmann equation without the elastic collisions and the transport theory of electrons in a phonon background. Conclusive comments are reported in Section 9, whilst an Appendix contains detailed information on the distribution functions obtained in this paper.

2. Outline of the problem

Consider a spatially homogeneous medium of FP with mass M, endowed with one excited internal energy level. We call $\Delta E > 0$ the gap of internal energy between the excited and the fundamental level. Through this medium we consider TP, endowed with mass m and charge $\mathcal{Q}$, which diffuse in the presence of an external electric field $\boldsymbol{E}$ and/or magnetic field $\boldsymbol{B}$. The TP are supposed to interact with the medium according to the following scheme

$$\mathrm{TP} + \mathrm{FP}_1 \rightleftharpoons \mathrm{TP} + \mathrm{FP}_2, \tag{2.1}$$

where FP_1 and FP_2 represent the fundamental and excited state of FP, whose number density will be denoted by $\mathcal{N}_1$ and $\mathcal{N}_2$, respectively. The number density n of TP is considered much lower than the number density $\mathcal{N} = \mathcal{N}_1 + \mathcal{N}_2$ of FP, so that, the medium can be modeled as a fixed background in thermodynamical equilibrium at the temperature T.

According to statistical mechanics we have

$$\frac{\mathcal{N}_2}{\mathcal{N}_1} = \exp\left(-\frac{\Delta E}{kT}\right). \tag{2.2}$$

In this case, the kinetic equations for the distribution function of TP $f \equiv f(\boldsymbol{x}, \boldsymbol{v}, t)$ can be written as follows

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f}{\partial \boldsymbol{x}} + \frac{\mathcal{Q}}{m}\Big(\mathbf{E} + \boldsymbol{v}\times\boldsymbol{B}\Big)\cdot\frac{\partial f}{\partial \boldsymbol{v}} = \left(\frac{\partial f}{\partial t}\right)^{\mathrm{el}}_{\mathrm{coll}} + \left(\frac{\partial f}{\partial t}\right)^{\mathrm{in}}_{\mathrm{coll}}. \tag{2.3}$$

The elastic collision integral is given by

$$\left(\frac{\partial f}{\partial t}\right)^{\mathrm{el}}_{\mathrm{coll}} = \int\int g\, I^{\mathrm{el}}(g, \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')\Big[f(\boldsymbol{v}')\mathcal{F}(\boldsymbol{w}') - f(\boldsymbol{v})\mathcal{F}(\boldsymbol{w})\Big]\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{\Omega}', \tag{2.4}$$

where

$$\mathcal{F} = \mathcal{N}\left(\frac{M}{2\pi kT}\right)^{3/2}\exp\left(-\frac{Mv^2}{2kT}\right), \tag{2.5}$$

and $I^{\mathrm{el}}(g, \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')$ is the elastic cross section, $g = |\boldsymbol{v} - \boldsymbol{w}|$ is the relative speed with $\boldsymbol{v}$ and $\boldsymbol{w}$ the velocities of the incoming particles, whereas the post-collision velocities are given by

$$\boldsymbol{v}' = \frac{1}{2(m+M)}\left(m\boldsymbol{v} + M\boldsymbol{w} + M g\boldsymbol{\Omega}'\right), \tag{2.6}$$

$$\boldsymbol{w}' = \frac{1}{2(m+M)}\left(m\boldsymbol{v} + M\boldsymbol{w} - M g\boldsymbol{\Omega}'\right). \tag{2.7}$$

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