



Statistical physics and fluctuations in ballistic non-equilibrium systems

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ARTICLE INFO

Article history:

Received 11 November 2008

Received in revised form 27 January 2009

Available online 28 February 2009

PACS:

05.70.Ln

Keywords:

Canonical ensemble

Nonequilibrium thermodynamics

Ballistic transport

Heat transport

ABSTRACT

In nonequilibrium systems in the ballistic transport regime, every point of the system contains particles arriving from different regions – each of them at different temperatures – and there are only few collisions, in such a way that equilibrium between the different populations will be reached very slowly. Here, we tentatively approach the local distribution function by a superposition of local-equilibrium distribution functions with different temperatures, corresponding to the different starting positions of the particles. In a second-order expansion, we find a distribution function which depends not only on the Hamiltonian H but also on H^2 , and we study the additional contribution to energy fluctuations.

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Some relevant nonequilibrium systems may be characterized by several different temperatures. For instance, in plasma physics, ions and electrons have their own temperatures; the same is true for lattice and electrons in metals under the action of a short and intense laser pulse, and for spins and the crystal lattice in nonequilibrium solids. These different temperatures arise from the inefficient energy exchange in collisions of particles with very different masses. In general, there are many definitions of nonequilibrium effective temperatures, which lead to the same result in equilibrium situations but to different results out of equilibrium [1–6]. One of the usual ways of defining these temperatures is based on the average energy of the corresponding degrees of freedom. Here, we assume an extreme situation, in which a nonequilibrium system is locally characterized by a distribution function which is the superposition of local equilibrium distribution functions characterized by their respective different temperatures.

The motivation of our work is to consider dilute systems with ballistic transport, which at every point contain particles coming from different positions, at different local temperatures. Since the collision rates are low, the system stays for a long time as a superposition of particles with different distribution functions related to their respective initial positions. In this case, the usual local-equilibrium approach is not valid, because the system cannot be divided in small subsystems in internal equilibrium. If, for convenience, the system is divided in small subsystems, each of them will contain particles coming from relatively long distances and their contribution should exhibit the non-local features of the state. Thus, the effective nonequilibrium entropy and nonequilibrium temperature in such situations must go beyond the local equilibrium approximation; this is a topic of interest in current nonequilibrium thermodynamics [7–12].

We assume that the distribution function at a given position \mathbf{r} is given by

$$\rho(H(q, p), \mathbf{r}) = \int \frac{e^{-H(q,p)/kT} f(T, \mathbf{r})}{Z} dT, \quad (1)$$

with q and p the positions and momenta of the N particles forming the system and where we call the integrating variable T statistical temperature, H is the Hamiltonian, referred to the particles whose position q is in the spatial volume between \mathbf{r}

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and $\mathbf{r} + d\mathbf{r}$, $f(T)$ is the probability distribution of T , and Z the partition function

$$Z(\mathbf{r}) = \int \int e^{-H(q,p)/kT} f(T, \mathbf{r}) dT d\Omega = \int B(H(q,p), \mathbf{r}) d\Omega, \quad (2)$$

where $d\Omega \simeq d^{3N} p d^{3N} q$, and where we define

$$B(H(q,p), \mathbf{r}) \equiv \int \exp(-H(q,p)/kT) f(T, \mathbf{r}) dT \quad (3)$$

as the nonequilibrium Boltzmann factor. In defining H , and performing the volume integral in (2), we are only considering the energy of the system over a volume ΔV in the surroundings of \mathbf{r} big enough to apply statistics and sufficiently small to consider that the distribution function defined in (1) will not change significantly, because we assume the short-ranged forces.

One should not confuse the statistical temperature T , that is an integrating variable, with the thermodynamical temperature $\partial S/\partial U$, that is a measurable value. In equilibrium, we have $f(T, \mathbf{r}) = \delta(T - T_0)$ with $T_0(\mathbf{r})$ the local equilibrium temperature, and these two variables are the same. In practice, $f(T, \mathbf{r})$ will be, if the system is not far from local equilibrium, a function with a peaked maximum at the local equilibrium temperature $T_0(\mathbf{r})$ and having a zero probability for the values far from it. For example, one could approximate it by using a Gaussian distribution centered at T_0 .

In fact, our proposal (1) and (2) can be considered as a plausible generalization of the local equilibrium approach to systems under ballistic regime. As a generalization of the local equilibrium theory, the distribution function $f(T, \mathbf{r})$ could vary from point to point.

It is important to note that in a local small volume near r the number of particles must be a greatly fluctuating value. Thus, a more general treatment should be based on an extension of the grand-canonical ensemble rather than on the canonical ensemble, used in (1) and (2). Therefore, the approach based on (1) and (2) is valid if the chemical potential of the particles is zero, for instance in photons or phonons that, on the other hand, are particles that behave in a ballistic way with ease and are the main carriers in several systems. To describe a system where the carriers are electrons, atoms or molecules, with chemical potential, the grand-canonical formulation should be used.

The transition from equilibrium to the model given by (1) and (2) could be described by the following scenario. If we suddenly impose a temperature difference to a system initially in equilibrium, and if this difference is low enough, it will evolve to a situation of local equilibrium, where the system is at internal equilibrium at every point but not globally. This could be achieved when the particles arriving at a point have a relaxation time short enough to let them relax before they move again. If we increase the temperature difference, the fluxes present in the system will also increase. The particles arriving at some point will not have time to change the form of their initial distribution. This situation makes that the distribution at every point will be the sum at different distributions of different temperatures.

Recently, Wilk and Włodarczyk [13] proposed a similar kind of behavior for the temperature. In it, the authors suppose that in nonequilibrium, the evolution of the system could be described by two different timescales. In the short timescale, the system is supposed to be in equilibrium, and one may define on it an equilibrium temperature. In the long timescale, this temperature fluctuates following a Fokker–Planck equation. From the evolution of the Fokker–Planck equation, the long time behavior of the system could be obtained.

Beck and Cohen [14] have proposed similar ideas for the entropy. They consider a system composed of multiple subsystems each of them in thermal equilibrium. In their theory, the temperature distribution $f(T)$ expresses the fact that temperature is a global function of the system that has different equilibrium values at different points but, in contrast to the situation we are analyzing here, their $f(T)$ does not depend on the position, as in our case, but it provides a global characterization of the system. The name “superstatistics” used by the authors describe the fact that the Boltzmann factor, with a form like (3), expresses a statistics over the classical statistics, and comes from a global fluctuation of a local intensive thermodynamic variable like, for example, temperature. The authors relate their superstatistics with the thermodynamical model of nonextensive entropy of Tsallis [15,16]. Tsallis and Souza [17] showed that if $\beta \equiv 1/kT$ is a fluctuating parameter following a γ distribution, Tsallis entropy is recovered, with $q = \langle \beta^2 \rangle / \langle \beta \rangle^2$, being the entropic index in Tsallis nonextensive thermodynamics.

In our formalism $f(T, \mathbf{r})$ has a similar meaning to these last proposals, but the weight factor in our proposal expresses that the system is a superposition, at every point, of equilibrium distributions and not a juxtaposition of equilibrium functions each one at different points, as in Refs. [14,15,17]. From this perspective, the Hamiltonian in our formalism has a local meaning. In “superstatistics” model this could be achieved by assigning a local meaning to the q factor, that is, to the parameter describing the fluctuation on the system.

Here, we assume that the interaction forces, present in the Hamiltonian, are short ranged; in the situation we are considering, the nonlocality comes from the long mean free path of the particles, as we are interested in ballistic transport. This justifies the interpretation of H as a local Hamiltonian. In contrast, in a general view of superstatistics one could consider long-ranged Hamiltonians, incorporating, for instance, gravitational interactions or critical points with long correlations. In this case, one could not properly define a local Hamiltonian, and the Hamiltonian should be the one for the whole system.

In this brief report we explore the consequences of a multiple statistical temperature distribution on the entropy, the thermodynamic temperature and the energy fluctuations. In order to integrate (1), we expand the Boltzmann factor

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