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Bose–Einstein condensation of bouncing balls

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ABSTRACT

Microscopic bouncing balls, i.e., particles confined within a positive one-half-dimensional gravitational potential, display Bose–Einstein condensation (BEC) not only in the thermodynamic limit but also in the case of a finite number of particles, and the critical temperature with a finite number of particles is higher than that in the thermodynamic limit. This system is different from the one-dimensional harmonic potential one, for which the standard result indicates that the BEC is not possible unless the number of particles is finite.

1. Introduction

Along with creating the Bose–Einstein statistics for the ideal boson gas, Einstein in 1925 showed that from a certain temperature on, the molecules condense without attractive forces [1], and it was named later as Bose–Einstein condensation (BEC). Seventy years after the prediction of Einstein, Bose–Einstein condensates formed by atomic gases confined in harmonic magnetic traps were observed at very low temperatures [2]. Inspired by either the theoretical modeling or the real experimental setup, theoreticians search in wider domains for the relation between BEC and the form of the potentials trapping the bosons. The well-studied situation is closely related to that of three-dimensional harmonic magnetic traps [3–5], where the mark of the transition temperature with a finite number of particles appears lower than that in the thermodynamic limit [6], and lowering the dimension increases the transition temperature, and therefore is favorable for BEC [3].

It is well-known that without any external field, the free bosons confined in a box in dimensions fewer than 3 will not condense. Starting from the one-dimensional gas of particles confined by a power-law potential $U(x) \sim |x|^{\eta}$, studies show that in the thermodynamic limit it will display BEC only if the potential power η satisfies $0 \prec \eta < 2$ [7]. In other words, the standard treatment indicates that the BEC is not possible for a one-dimensional harmonic potential $\eta = 2$, even it can be defined with a finite number of particles [3,7].

Once physical systems are studied on the Earth, the influence of the gravitational field cannot be overlooked. However, even with the lack of quantitative results, a qualitative analysis was made forty-one years ago [8], showing that in the presence of the gravitational field, the inhomogeneous boson gas can condense in one and two dimensions. It is therefore understandable that the gravitational field added into the independent free bosons in a three-dimensional box can quantitatively affect the critical temperature of BEC [9]. In the present note, we give quantitative results for BEC in a

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one-dimensional half-space decorated with a gravitational field, with emphasis on possible effects resulting from the finite number of particles.

In Section 2, a semiclassical treatment of the one-dimensional noninteracting bosons in the presence of a gravitational field is given, which is applicable in the thermodynamic limit. In Section 3, the effect of the finite number of particles is discussed. This article is closed with a brief conclusion in Section 4.

2. A semiclassical treatment of one-dimensional BEC in a gravitational field

In Bose–Einstein statistics, the average number of particles in an energy eigenstates E_n is given by the Bose–Einstein distribution,

$$N_n = \frac{1}{\exp((E_n - \mu)/k_B T) - 1}, \tag{1}$$

where μ is the chemical potential and k_B is Boltzmann's constant. The chemical potential μ is determined by the constraint that the total number of particles in the system is N,

$$N = \sum_{n=0}^{\infty} N_n. \tag{2}$$

The particles of mass M are confined by a gravitational potential that is given by

$$U(x) = \begin{cases} Mgx, & (x > 0) \\ \infty, & (x \leq 0). \end{cases} \tag{3}$$

So, the particle is actually a classical bouncing ball that moves in a positive half-space in one dimension. For the case where the temperature $k_B T$ is much higher than that for any two adjoining quantum levels, i.e., $k_B T \gg \varepsilon_2 - \varepsilon_1$, the system can be well described as a continuum of energy levels plus a separate ground state. The density of states is then given by

$$\rho(\varepsilon) = \frac{1}{h} \int \mathrm{d}x \mathrm{d}p \delta(H - \varepsilon), \tag{4}$$

where $H = p^2/(2M) + U(x)$ is the Hamiltonian for the particle. Then Eq. (4) becomes

$$\rho(\varepsilon) = \frac{\sqrt{2M}}{h} \int_0^{l(\varepsilon)} \frac{\mathrm{d}x}{\sqrt{\varepsilon - U(x)}} = \frac{2}{hg}\sqrt{\frac{2}{M}}\sqrt{\varepsilon}, \tag{5}$$

where $l(\varepsilon) = \varepsilon/(Mg)$ is the maximum height for classical particles with energy ε. The total number of particles satisfies, from Eq. (2),

$$N = N_0 + \int_0^{\infty} \frac{\rho(\varepsilon)\mathrm{d}\varepsilon}{\exp((\varepsilon - \mu)/k_B T) - 1}, \tag{6}$$

where N_0 is the number of particles in the ground state $\varepsilon = 0$. For a given temperature, the maximum number of particles accommodated in excited states is reached when $\mu = 0$. Then the critical temperature T_C can be determined using the following equation:

$$N = \int_0^{\infty} \frac{\rho(\varepsilon)\mathrm{d}\varepsilon}{\exp(\varepsilon/k_B T_C) - 1}, \tag{7}$$

which can be rewritten as

$$T_C = \left(\frac{MghN}{2.612 k_B^{3/2} \sqrt{2\pi M}} \right)^{2/3} = 1.226 N^{2/3} \left(\frac{Mg^2 \hbar^2}{2} \right)^{1/3} \frac{1}{k_B}. \tag{8}$$

To see quantitatively at what number N the molecules start to condense, we obtain some numerical results. For an air molecule of average mass $M = 4.82 * 10^{-26}$ kg, with the requirement that $T_C = 300$ K, BEC starts once $N \geq 1.28 * 10^{15}$, while for a hydrogen gas molecule of mass $M = 3.35 * 10^{-27}$kg, $N \geq 4.84 * 10^{15}$. At the extreme, $N(T_C) = 1$ gives from Eq. (7) $T_C = 25.5$ nK and 10.5 nK for air and hydrogen gas respectively, and no molecule can be excited as long as $T \prec T_C$. These numerical results show that the continuum treatment is applicable once the critical temperature is much higher than 10 nK, the temperature corresponding to the energy difference between the ground state and the first excited state. In other words, at temperature lower than 10 nK, a few particles are sufficient for condensation. So, the finite number effects can never be overlooked at low temperature. Moreover, we will see in the next section an interesting behaviour: the critical temperature is higher than that in the thermodynamic limit.