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A versatile entropic measure of grey level inhomogeneity

Ryszard Piasecki *

Faculty of Chemistry, University of Opole, Oleska 48, 45-052 Opole, Poland

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An entropic measure for the analysis of grey level inhomogeneity (GLI) is proposed as a function of length scale. It allows us to quantify the statistical dissimilarity of the actual macrostate and the maximizing entropy of the reference one. The maximums (minimums) of the measure indicate those scales at which higher (lower) average grey level inhomogeneity appears compared to neighbour scales. Even a deeply hidden statistical grey level periodicity can be detected by the equally distant minimums of the measure. The striking effect of multiple intersecting curves (MICs) of the measure has been revealed for pairs of simulated patterns, which differ in shades of grey or symmetry properties only. In turn, for evolving photosphere granulation patterns, the stability in time of the first peak position has been found. Interestingly, the third peak is dominant at initial steps of the evolution. This indicates a temporary grouping of granules at a length scale that may belong to the mesogranulation phenomenon. This behaviour has similarities with that reported by Consolini, Berrilli et al. [G. Consolini, F. Berrilli, A. Florio, E. Pietropaolo, L.A. Smaldone, Astron. Astrophys. 402 (2003) 1115; F. Berrilli, D. Del Moro, S. Russo, G. Consolini, Th. Straus, Astrophys. J. 632 (2005) 677] for binarized granulation images of a different data set.

1. Introduction

The morphological features of complex materials are vital for modelling and predicting their macroscopic properties, for instance, effective conductivity [1–4]. From binary micrographs of such materials, the quantitative characterization of the spatial distribution of pixels in some cases allows one to correlate their properties and internal structure attributes. As the white and black pixels are finite size objects (FSOs), one can assume that the latter represent hard-core particles. When points approximate particles, the system can be described by a complete set of n-point correlation functions [1]. In practice, their calculation is rather difficult, with the exception of two-point correlation function.

The much simpler statistical method of finding different length scale dependent morphological features for disordered systems gives normalized information entropy [5] or cluster diversity and cluster entropy [6,7]. The length scale is defined by the side size of the sampling square cell. However, the measures employ a set of discrete probabilities, whose statistical meaning weakens at large length scales [5] or given low/high lattice site occupation probability [6,7]. The spatial inhomogeneity for FSO distributions can be also evaluated by (i) a simple *mathematical* statistical measure, which compares a certain random variable with its expected value (its final modification is given by Eq. (2) in Ref. [8]) or (ii) a *physical* entropic measure that quantifies a *deviation* per cell of a given configurational macrostate from the reference one describing the most uniform FSO distribution (cf. Eq. (4) in Ref. [9]). In the latter approach a difference of the corresponding configurational entropies is used. For $L \times L$ patterns partitioned in non-overlapping cells, the mathematical measure is limited to length scales being integer divisors of L; otherwise only its approximated values are available even for a periodically repeated

* Tel.: +48 77 452 7116; fax: +48 77 452 7101.
E-mail address: piaser@uni.opole.pl.

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initial arrangement. The physical entropic measure, due to its conservation property, is free of such limitations and thus it can be named as the exact one. It can detect even subtle self-similarity traces in model surface microstructures [10]; in Eq. (3) the '+' sign before the second term should be replaced by opposite one, '−'. In turn, when a sliding cell sampling (SCS) is applied, all length scales are also allowed for the mathematical measure. The last two measures initially slightly differ while at the other scales they correlate well. The maximal range of appropriate length scales, even for an assumed minimal number of sampled cells of order 1000, is equal to $L - 31$ according to a simple condition for good cell statistics given in Ref. [8].

On the other hand, experimental data and the results of theoretical simulations are frequently presented through greyscale or colour images. The latter can be also carefully converted into 8-bit greyscale images with 256 shades of grey between black and white colours [11]. It is obvious that more complex data can be encoded with this type of pattern compared to 1-bit binary ones. Thus, the revealing of any length scale dependent characteristic features of grey level patterns is of some importance. The grey level extension proposed here is a natural *completion* of the binary entropic measure [9,10]. Instead of the spatial degrees of freedom involved formerly, the extended measure considers specific degrees of freedom. Those relate to possible distributions, under certain conditions, of every cell sum of grey level values inside the corresponding cell at a given length scale. Then two configurational macrostates, defined in the next section, are compared per cell: the one obtained on the basis of the existing pattern and the calculated reference one related to the most uniform distribution of grey level values. Thus, the present entropic measure quantifies a kind of average grey level inhomogeneity over a range of length scales.

2. The extended entropic measure

Let us assume that the initial pattern is a representative one for the investigated physical system. Instead of its standard partitioning employed in Refs. [9,10] here we use the SCS method; see for instance Ref. [8]. For a given 8-bit greyscale image of size $L \times L$ there are $\kappa(k) = [L - k + 1]^2$ allowed positions of the sliding cell of size $k \times k$. When the sampled cells are placed in a non-overlapping manner, the auxiliary pattern $L_a \times L_a$, where $L_a \equiv [L - k + 1]k$, obtained in this way reproduces the general structure of the initial one; cf. Fig. 1(b). The SCS procedure provides local sums $G_i(k)$ of grey level values for each sampled ith cell of the initial pattern. At each length scale $1 \leq k \leq L$ the only restriction for local sums is the natural constraint given by

$$\sum_{i=1}^{\kappa} G_i(k) = G(k), \tag{1}$$

where $G(k)$ stands for the length scale dependent total sum of grey level values.

For a given configuration at scale k, the actual macrostate, AM(k), can be defined by the corresponding set, $\{G_i(k)\}_{AM} \equiv (G_1(\mathrm{AM}), \ldots, G_\kappa(\mathrm{AM}))$. Inside every cell of size $k \times k$ all unit cells each of size 1×1 can be numbered in sequence $1, 2, \ldots, k^2$. When for an 8-bit greyscale image all 256 shades of grey, i.e., from the range (0–255) [11] are taken into account, then certain of the unit cells can be occupied by zero valued grey level. Within the simplest approach we consider all possible order dependent partitions, allowing some of the parts to be zero, of obtained local sum $G_i(k)$ over the k^2 unit cells inside the ith cell for $i = 1, 2, \ldots, \kappa$. In mathematics this is sometimes referred to as a *weak composition* [12]. Then we calculate the number $\Omega_{gr}(k)$ of realizations of AM(k), i.e., the number of the appropriate configurational microstates that is the product of the ways that each of sampled cells can be populated under the above conditions:

$$\Omega_{\mathrm{gr}}(k, G) = \prod_{i=1}^{\kappa} \binom{G_i + k^2 - 1}{k^2 - 1}. \tag{2}$$

Now, we make use of a microcanonical entropy, $S_{gr}(k) = k_B \ln \Omega_{gr}(k)$, where the Boltzmann constant will be set as $k_B = 1$ for convenience. It is worth noticing that the entropic approach has already been utilized for binary patterns [9,10]. To obtain at a given length scale k the highest possible value of the entropy, $S_{\mathrm{gr,max}}(k) = \ln \Omega_{\mathrm{gr,max}}(k)$, we need a reference macrostate, RM(k), ensuring the maximal number $\Omega_{\mathrm{gr,max}}(k)$ of its realizations. Physically, RM(k) corresponds to the most *uniformly* distributed local sums $G_i(k)$ under the constraint (1). Thus, it can be described by the appropriate set, $\{G_i(k)\}_{RM} \equiv (G_1(RM), \ldots, G_\kappa(RM))$, where each pair $i \neq j$ of sampled cells fulfils the simple condition $|G_i(k) - G_j(k)| \leq 1$. The validity of this simple condition has been checked for model systems by computer simulations. For the simplest case of only two cells it is confirmed by elementary mathematical considerations. Therefore, in agreement with statistical physics, the maximal number of the proper microstates can be written as

$$\Omega_{\mathrm{gr,max}}(k, G) = \binom{G_0 + k^2 - 1}{k^2 - 1}^{\kappa - R_0} \binom{G_0 + k^2}{k^2 - 1}^{R_0}, \tag{3}$$

where $R_0(k) = G(k) \mod \kappa$ and $G_0(k) = [G(k) - R_0(k)]/\kappa$. Every microstate of the $\Omega_{\mathrm{gr,max}}$ set represents a reference macrostate $\{G_i \in (G_0, G_0 + 1)\}_{RM}$ with $\kappa - R_0$ and R_0 number of cells with sums G_0 and $G_0 + 1$ of grey levels, respectively.