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A unified model for Sierpinski networks with scale-free scaling and small-world effect

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ABSTRACT

In this paper, we propose an evolving Sierpinski gasket, based on which we establish a model of evolutionary Sierpinski networks (ESNs) that unifies deterministic Sierpinski network [Z.Z. Zhang, S.G. Zhou, T. Zou, L.C. Chen, J.H. Guan, Eur. Phys. J. B 60 (2007) 259] and random Sierpinski network [Z.Z. Zhang, S.G. Zhou, Z. Su, T. Zou, J.H. Guan, Eur. Phys. J. B 65 (2008) 141] to the same framework. We suggest an iterative algorithm generating the ESNs. On the basis of the algorithm, some relevant properties of presented networks are calculated or predicted analytically. Analytical solution shows that the networks under consideration follow a power-law degree distribution, with the distribution exponent continuously tuned in a wide range. The obtained accurate expression of clustering coefficient, together with the prediction of average path length reveals that the ESNs possess small-world effect. All our theoretical results are successfully contrasted by numerical simulations. Moreover, the evolutionary prisoner's dilemma game is also studied on some limitations of the ESNs, i.e., deterministic Sierpinski network and random Sierpinski network.

1. Introduction

In the last few years, complex networks have attracted a growing interest from a wide circle of researchers [1–4]. The reason for this boom is that complex networks describe various systems in nature and society, such as the World Wide Web (WWW), the Internet, collaboration networks, and sexual network, and so on. Extensive empirical studies have revealed that real-life systems have in common at least two striking statistical properties: power-law degree distribution [5], small-world effect [6] including small average path length (APL) and high clustering coefficient. In order to mimic real-word systems with above mentioned common characteristics, a wide variety of models have been proposed [1–4]. At present, it is still an active direction to construct models reproducing the structure and statistical characteristics of real systems.

In our previous papers, on the basis of the well-known Sierpinski fractal (or Sierpinski gasket), we have proposed a deterministic network called deterministic Sierpinski network (DSN) [7], and a stochastic network named random Sierpinski network (RSN) [8], respectively. Both the DSN and RSN possess good topological properties observed in some real systems. In this paper, we suggest a general scenario for constructing evolutionary Sierpinski networks (ESNs) controlled by a parameter q. The ESNs can also result from Sierpinski gasket and unify the DSN and RSN to the same framework, i.e., the DSN and RSN are special cases of RSNs. The ESNs have a power-law degree distribution, a very large clustering coefficient, and a small

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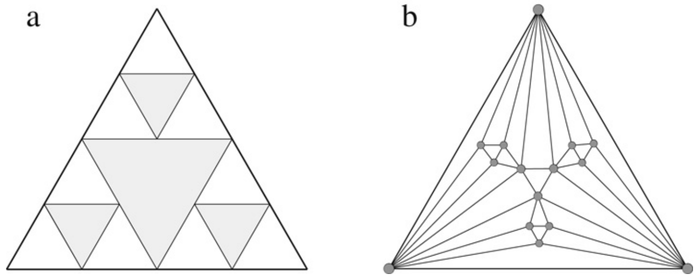

Fig. 1. The first two stages of construction of the Sierpinski gasket (a) and its corresponding network (b).

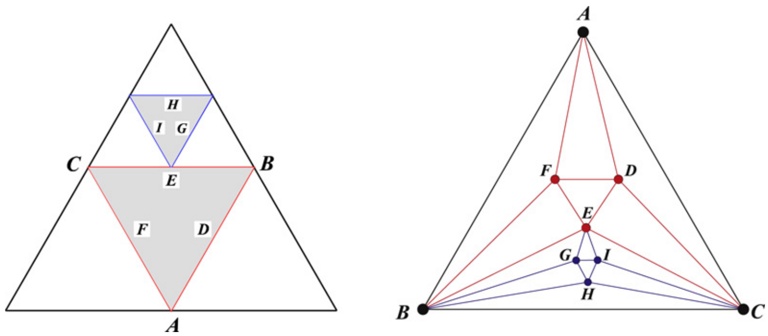

Fig. 2. (Color online) The sketch maps for the construction of random Sierpinski gasket (left panel) and its corresponding network (right panel).

intervertex separation. The degree exponent of ESNs is changeable between 2 and 3. Moreover, we introduce a generating algorithm for the ESNs which can realize the construction of our networks. In the end, the cooperation behavior of the evolutionary prisoner's dilemma game on two limiting cases (i.e., DSN and RSN) of the ESNs is discussed.

2. Brief introduction to deterministic and random Sierpinski networks

We first introduce Sierpinski gasket, which is also known as Sierpinski triangle. The classical Sierpinski gasket denoted as S_t after t generations, is constructed as follows [9,10]: start with an equilateral triangle, and denote this initial configuration as S_0. Perform a bisection of the sides forming four small copies of the original triangle, and remove the interior triangles to get S_1. Repeat this procedure recursively in the three remaining copies to obtain S_2, see Fig. 1(a). In the infinite t limit, we obtain the famous Sierpinski gasket S_t. From Sierpinski gasket we can easily construct a network, called deterministic Sierpinski network, with sides of the removed triangles mapped to nodes and contact to edges between nodes [7]. For uniformity, the three sides of the initial equilateral triangle at step 0 also correspond to three different nodes. Fig. 1(b) shows a network based on S_2.

Analogously, one can construct the random Sierpinski network [8] derived from the stochastic Sierpinski gasket, which is a random variant of the deterministic Sierpinski gasket. The initial configuration of the random Sierpinski gasket is the same as the deterministic Sierpinski triangle. Then in each of the subsequent generations, an equilateral triangle is chosen randomly, for which bisection and removal are performed to form three small copies of it. The sketch map for the random fractal is shown in the left panel of Fig. 2. From this fractal we can easily establish the random Sierpinski network with sides of the removed triangles mapped to nodes and contact to links between nodes. The right panel of Fig. 2 gives a network derived from the random Sierpinski gasket.

3. Unifying model and its iterative algorithm

In this section, we introduce an evolving unified model for the deterministic and random Sierpinski networks. First we give a new variation, called evolving Sierpinski gasket (ESG), for the Sierpinski gasket. The initial configuration of the ESG is the same as the deterministic Sierpinski gasket. Then in each of the subsequent generations, for each equilateral triangle, with probability q, bisection and removal are performed to form three small copies of it. In the infinite generation limit, the ESG is obtained. In a special case $q = 1$, the ESG is reduced to the classic deterministic Sierpinski gasket. If q approaches but is not equal to 0, it coincides with the random Sierpinski gasket described in Ref. [8]. The proposed unified model is derived from this ESG: nodes represent the sides of the removed triangles and edges correspond to contact relationship. As in the construction of the deterministic and random Sierpinski networks [7,8], the three sides of the initial equilateral triangle (at step 0) of the ESG are also mapped to three different nodes.