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Granular rough sphere in a low-density thermal bath

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ABSTRACT

We study the stationary state of a rough granular sphere immersed in a thermal bath composed of point particles. When the centre of mass of the sphere is fixed the stationary angular velocity distribution is shown to be Gaussian with an effective temperature lower than that of the bath. For a freely moving rough sphere coupled to the thermostat via inelastic collisions we find a condition under which the joint distribution of the translational and rotational velocities is a product of Gaussian distributions with the same effective temperature. In this rather unexpected case we derive a formula for the stationary energy flow from the thermostat to the sphere in accordance with the Fourier law.
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1. Introduction

The study of the dynamics of a tracer particle is a classical problem of nonequilibrium statistical mechanics. The object is to determine the evolution of the state of a single particle resulting from interaction with the surrounding medium. Such a study can yield valuable information on the effects of many-body dynamics. Moreover, the relative simplicity of the problem creates opportunities for precise theoretical predictions. A number of works aimed at understanding the dynamics of fluidized granular media have recently been produced [1]. In particular, the evolution of a granular sphere immersed in a granular medium homogeneously cooling down has been discussed [2] as well as Brownian motion in a granular fluid [3]. The case of an impurity put in a vibrating low-density granular system has also been studied [4].

An interesting qualitative question, related to the effects of inelastic collisions taking place in granular fluids, is that of the resulting structure of the distribution function when different kinds of degrees of freedom are present. A gas of rough spheres with both translational and rotational degrees of freedom has been recently examined from this point of view [5]. The main prediction, based on numerical studies and approximate analytic arguments, is that dissipative collisions induce statistical dependence between orientations of the angular and translational velocities.

In the present paper we address an analogous question in an even simpler situation of a single tracer granular rough sphere suffering inelastic collisions with point masses forming a low-density thermal bath. Our object is to find out what kind of stationary state can result from a dissipative coupling to a thermostat. In the case of elastic collisions the particle would eventually attain equilibrium at the temperature of the bath. But the nature of the asymptotic stationary state in which there is a constant dissipative heat flow from the thermostat to the tracer particle remains a largely open question of fundamental interest.

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Exact results have been derived for a smooth hard sphere where inelastic collisions could influence only translational motion. It turned out that at the level of the Boltzmann kinetic theory the stationary velocity distribution had the form of a Maxwell distribution with an effective temperature lower than that of the thermostat [6]. In one dimension one could even rigorously solve the initial value problem deriving, in particular, the exact dependence of the diffusion coefficient on dissipation [7]. In the case of purely translational degrees of freedom the appearance of a Gaussian distribution has been shown to follow from the equivalence between the Boltzmann equation for a granular tracer particle suffering inelastic collisions and the Boltzmann equation for an elastic tracer particle with a suitably modified mass [2].

This remarkable property also occurs if the test particle is rough but has a fixed mass centre and thus only rotational degrees of freedom [8] (an original derivation of this fact is presented in Section 3). The stationary angular velocity is then again Gaussian with an effective temperature lower than that of the thermostat.

In the present paper we extend the study of stationary states at the level of Boltzmann's kinetic theory to the case of a tracer rough sphere whose translational and rotational motions are both influenced by inelastic collisions (the distribution of kinetic energy in a granular gas composed of rough spheres has been discussed in [9,10]). In Section 2 we describe the model. Section 3 contains the description of our method first illustrated on simple situations where only one type of degrees of freedom is present. We then turn to the general case and show that, when the restitution coefficients for normal and tangential relative velocities obey a specific relation (37), the joint velocity distribution becomes a product of two Maxwell distributions for the angular and translational velocities corresponding to the *same* effective temperature. It turns out that the derived relation may be fulfilled only if the restitution coefficient relevant for rotational motion is larger than that linked to the motion of the mass centre.

The occurence of a stationary factorized Gaussian distribution for the two types of degrees of freedom inelastically excited by collisions is quite remarkable and, in view of the results obtained for a gas of granular rough spheres [5], rather unexpected. The main result of Section 3 is obtained by using an appropriate change of integration variables in the gain term of the Boltzmann equation (the method generalizes that used in Ref. [2]).

The heat flux that maintains the test particle in a stationary state is calculated in Section 4. It obeys the analogue of Fourier law with a thermal conductivity proportional to the temperature jump between the sphere and the thermostat, as in generic hydrodynamic theories. In Section 5 we briefly comment on the possible structure of the joint velocity distribution when the values of the restitution coefficients are not related by the equation derived in Section 3. We expect that a typical case would involve a statistical relationship between the angular and translational velocities of the sphere.

2. The model

For the sake of simplicity the thermal bath particles are supposed to be point masses m performing purely translational motion. Their distribution in the one-particle phase space is the product of a uniform spatial density ρ and a Maxwell velocity distribution $\phi_T(\mathbf{v}; m)$ corresponding to temperature T

$$\phi_T(\mathbf{v}; m) = \left(\frac{m}{2\pi k_B T}\right)^{D/2} \exp\left[-\frac{m\mathbf{v}^2}{2k_B T}\right] \tag{1}$$

D is the dimension of the space ($D = 2$ or 3), and k_B is Boltzmann's constant.

The rough sphere is supposed to have mass M, radius R, moment of inertia I, and to move with translational velocity $\mathbf{V}$, and angular velocity $\mathbf{\Omega}$. Thus its total kinetic energy equals

$$E(\mathbf{V}, \mathbf{\Omega}) = \frac{1}{2}M\mathbf{V}^2 + \frac{1}{2}qMR^2\mathbf{\Omega}^2 \tag{2}$$

where $q = I/MR^2$ is a number reflecting the mass density distribution inside the sphere (disk).

2.1. Collisional laws

Consider a binary collision between the rough sphere and a point particle of the thermostat. The instantaneous collisional transformation of velocities

$$(\mathbf{V}, \mathbf{\Omega}, \mathbf{v}) \to (\mathbf{V}^*, \mathbf{\Omega}^*, \mathbf{v}^*) \tag{3}$$

is conveniently described with the help of the unit vector $\hat{\mathbf{n}}$ along the line segment from the centre of the sphere to the point of impact.

The linear velocity of the point at the surface of the sphere hit by the thermostat particle is $(\mathbf{V} + R\mathbf{\Omega} \times \hat{\mathbf{n}})$. The relative velocity at which the particle approaches the impact point is thus

$$\mathbf{g} = \mathbf{v} - \mathbf{V} - R\mathbf{\Omega} \times \hat{\mathbf{n}}. \tag{4}$$

In what follows, we will use the notations

$$\mathbf{A}_n = (\mathbf{A} \cdot \hat{\mathbf{n}})\,\hat{\mathbf{n}}, \quad \text{and} \quad \mathbf{A}_t = \mathbf{A} - \mathbf{A}_n = \hat{\mathbf{n}} \times (\mathbf{A} \times \hat{\mathbf{n}}) \tag{5}$$

for the normal and tangential components of any vector $\mathbf{A}$.

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