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Stability analysis solutions for nonlinear three-dimensional modified Korteweg–de Vries–Zakharov–Kuznetsov equation in a magnetized electron–positron plasma

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HIGHLIGHTS

- The hydrodynamic model is applied to three-dimensional magnetized electron–positron plasma waves.
- New exact solutions for the modified Korteweg–de Vries–Zakharov–Kuznetsov equation, wave solutions, modified direct algebraic method.
- We will present three traveling-wave solutions to modified Korteweg–de Vries–Zakharov–Kuznetsov equation.
- We discussed the stability analysis for these solutions.

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ABSTRACT

The nonlinear three-dimensional modified Korteweg–de Vries–Zakharov–Kuznetsov (mKdV–ZK) equation governs the behavior of weakly nonlinear ion-acoustic waves in magnetized electron–positron plasma which consists of equal hot and cool components of each species. By using the reductive perturbation procedure leads to a mKdV–ZK equation governing the oblique propagation of nonlinear electrostatic modes. The stability of solitary traveling wave solutions of the mKdV–ZK equation to three-dimensional long-wavelength perturbations is investigated. We found the electrostatic field potential and electric field in form traveling wave solutions for three-dimensional mKdV–ZK equation. The solutions for the mKdV–ZK equation are obtained precisely and efficiency of the method can be demonstrated.

1. Introduction

Electron–positron (EP) plasmas are frequently encountered in the early Universe and relevant in many space environments. The EP occurrence is in active galactic nuclei, the pulsar magnetosphere, accretion disks, pulsar magnetospheres, neutron stars, cosmic solar flares, black hole magnetospheres [1–3]. The physics properties of EP plasmas are different from that of an electron–ion plasma. The modulational instability of the wave equation of ultra-intense linearly polarized laser pulse propagating in EP plasmas was investigated. The magnetohydrodynamics of an electron–positron plasma is expected to show some new physical results, which could be significantly different from those of an electron–ion plasma [4].

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The ion-acoustic solitary wave (IASW) is one of the solitary waves that were studied with a long history and their numerical simulations were developed. The IASW are a kind of important nonlinear structures in plasmas. The experimental observations on the reflection and refraction of IASW were performed. Finite amplitude IASW propagating obliquely to an external magnetic field in a plasma without dust particles were investigated [5]. The nonlinear IASW propagating obliquely to the external magnetic field in a magnetized dusty plasma were studied [6–8]. The Korteweg–de Vries–Zakharov–Kuznetsov equations for a mixture of hot isothermal, warm adiabatic fluid and cold immobile background species in a magnetized plasma by the reductive perturbation technique was derived, and electron-acoustic, ion-acoustic and dust-acoustic solitons were investigated [9].

The Zakharov–Kuznetsov (ZK) equation is a very attractive model equation for the study of vortices in geophysical flows. The ZK equation appears in many areas of physics, applied mathematics and engineering [10,11]. The ZK equation governs the behavior of weakly nonlinear ion-acoustic waves in a plasma comprising of cold ions and hot isothermal electrons in the presence of a uniform magnetic field [12,13]. A three-dimensional extended Zakharov–Kuznetsov (EZK) equation was derived for small but finite amplitude nonlinear Langmuir structures. The EZK equation was solved analytically and the features of the nonlinear excitations were investigated numerically [14].

The weakly nonlinear electron acoustic wave in a two electron component magnetized plasma was studied [15–17]. The Korteweg–de Vries–Zakharov–Kuznetsov (KdV–ZK) equation leading to plane and ellipsoidal soliton solutions to explain some of the two dimensional features of solitary wave observations was derived [18]. A modified KdV–ZK equation governing the oblique propagation of nonlinear electrostatic modes was derived using the reductive-perturbation technique. Soliton amplitudes were studied as a function of plasma parameters such as the particle number densities and the temperatures. Such results may be of relevance to the magnetosphere of pulsars [19]. The Zakharov–Kuznetsov–Burgers (ZKB) equation was derived from the ion continuity equation, ion momentum equation with kinematic viscosity among ions fluid, electrons and positrons having kappa distribution together with the Poisson equation [20]. The theory of planar dynamical systems was applied to carry out a qualitative analysis to the planar dynamical system corresponding to the bounded traveling wave solution of the ZKB equation [21]. The nonlinear self-adjointness condition for the ZKB equation was established and subsequently used to construct simplified but infinitely many nontrivial and independent conserved vectors [22–25].

The electromagnetic solitary structures are significantly modified by the effects of degenerate electron and positron pressures. The applications of the results in an EP plasma medium occur in compact astrophysical objects. Traveling wave analysis is given in Ref. [26] for the ZK equation. Soliton solutions were derived using the improved modified extended tanh-function method [27,28]. One-dimensional soliton, apparently inelastic, periodic solutions and N-soliton solutions have been obtained. The auxiliary equation method and the direct Hirota bilinear method were applied to the quantum ZK equation in Refs. [29–32].

This paper is organized as follows: An introduction is given in Section 1. In Section 2, the problem formulation to derive the nonlinear three-dimensional modified KdV–ZK equation is formulated. In Section 3, the conservation laws for the mKdV–ZK equation of weakly nonlinear ion-acoustic waves in a magnetized electron–positron plasma are found. In Section 4, Hamiltonian system for the momentum and the sufficient condition for the soliton solutions stability are given. In Section 5, the electric field potential and electric field in form traveling wave solutions of the mKdV–ZK equation are obtained and analyzed. Finally the paper ends with a conclusion given in Section 6.

2. Problem formulations

Consider the waves propagation in a three-dimensional homogeneous magnetized, electron–positron plasma, consisting of equal hot and cool components of each species. The dynamics of the cooler adiabatic species are governed by the fluid equations as

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{u}_\alpha) = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}_\alpha}{\partial t} + (\mathbf{u}_\alpha \cdot \nabla)\mathbf{u}_\alpha = -\frac{1}{n_\alpha m_\alpha}\nabla p_\alpha - \frac{q_\alpha}{m_\alpha}\nabla\phi + \Omega_\alpha \mathbf{u}_\alpha \wedge \mathbf{e}_x, \tag{2}$$

$$\frac{\partial p_\alpha}{\partial t} + \mathbf{u}_\alpha \cdot \nabla p_\alpha + \gamma_\alpha p_\alpha (\nabla \cdot \mathbf{u}_\alpha) = 0, \tag{3}$$

$$\epsilon_0 \nabla^2 \phi + \sum_\alpha n_\alpha q_\alpha + \sum_\beta N_\beta q_\beta \exp\left(\frac{-q_\beta \phi}{kT_\beta}\right) = 0, \tag{4}$$

$$n_{eh} = N_h \exp\left(\frac{e\phi}{kT_h}\right), \qquad n_{ph} = N_h \exp\left(\frac{-e\phi}{kT_h}\right) \tag{5}$$

where T_h and N_h are hot electrons and positrons with equal temperatures and equilibrium densities, n_{eh} and n_{ph} are the density of the hot electrons and positrons, ϕ is the electrostatic potential, $\mathbf{u}_\alpha$ and p_α are the fluid velocities and pressures, q_α and q_β are the charges of the cool and hot species, $m = m_e = m_p$ is the common mass of the electrons and the positrons,