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Option pricing for stochastic volatility model with infinite activity Lévy jumps

Xiaoli Gong *, Xintian Zhuang

School of Business Administration, Northeastern University, Shenyang, 110169, China

HIGHLIGHTS

- Stochastic volatility models and infinite activity Lévy processes are combined.
- The leptokurtosis and heteroskedasticity properties in stock returns are captured.
- Apply intelligent optimization Differential Evolution algorithm to parameters calibration.
- Researches illustrate the superiority of tempered stable distribution in stochastic volatility model.

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ABSTRACT

The purpose of this paper is to apply the stochastic volatility model driven by infinite activity Lévy processes to option pricing which displays infinite activity jumps behaviors and time varying volatility that is consistent with the phenomenon observed in underlying asset dynamics. We specially pay attention to three typical Lévy processes that replace the compound Poisson jumps in Bates model, aiming to capture the leptokurtic feature in asset returns and volatility clustering effect in returns variance. By utilizing the analytical characteristic function and fast Fourier transform technique, the closed form formula of option pricing can be derived. The intelligent global optimization search algorithm called Differential Evolution is introduced into the above highly dimensional models for parameters calibration so as to improve the calibration quality of fitted option models. Finally, we perform empirical researches using both time series data and options data on financial markets to illustrate the effectiveness and superiority of the proposed method.

1. Introduction

Recently, considerable researches have been conducted on option pricing in stochastic volatility models. The improvements of our presented model over the prevailing are the introduction of infinite jump behaviors for underlying asset returns which are modeled by infinite activity Lévy processes, where they can capture asymmetry, leptokurtosis and thicker tail properties in returns in addition to characterizing persistence effect and heteroskedasticity effect in volatility. The Lévy processes driven stochastic volatility models incorporate jumps and stochastic volatility simultaneously, which capture not only jumps but also stochastic volatility in stock price dynamics. The Lévy process can be an infinite activity process that generates an infinite number of jumps within any time interval, based on which the closed form option pricing formula can be derived due to the attainability of analytical characteristic functions, or it can be a finite activity process

* Corresponding author. Tel.: +86 24 83679139.
E-mail address: qdgongxiaoli@126.com (X. Gong).

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such as compound Poisson processes. In this work, we specially pay attention to the case where the Lévy jumps numbers are infinite to capture both large and small jumps in stock returns.

The traditional B–S model assumes the dynamics of underlying asset returns follow a geometric Brownian motion with constant drift and volatility parameters, which contradicts the phenomenon persistently observed in financial markets. Several stylized facts about asset returns distributions are widely accepted which include asymmetry, leptokurtosis and thicker tail nature than Gaussian distributions [1,2]. Moreover, there is abundant evidence that volatility exhibits clustering and heteroskedasticity effect, leading to stochastic jumps in stock prices. It has been demonstrated by Ait-Sahalia and Jacod [3], Klingler [4] that stochastic volatility and jumps are inherent components of the stock price dynamics that play important roles in the explanation of the implied volatility smile in options. Many alternative models are developed to reflect the intrinsic characteristics of asset returns and volatility smile effects of option prices, as shown by Kou [5], Mozumder [6], Shi [7], and Abdelrazeq [8]. Therefore, when constructing models to price options, it is necessary to incorporate both stochastic volatility and jumps. Just as what Carr and Wu [9] have addressed that infinite activity Lévy processes can capture both large jumps and small jumps in stock price dynamics that are consistently inspected in financial markets.

The alternative approach to solve the heteroskedasticity and volatility clustering effect for returns variance is by applying stochastic volatility models [10,11]. Even though plentiful work has been done on jump-diffusion stochastic volatility models see, for example, Todorov [12], Andreas et al. [13], Huang et al. [14], they feature a counter-factual assumption that jumps rarely occur. Bates [15] refined the Heston [16] model through adding a normally distributed compound Poisson jump for stock price dynamics, with the returns variance following the Cox–Ingersoll–Ross (CIR) mean reverting process employed in Schoutens [17]. Hence, it is reasonable to substitute the finite jump components in the Bates model into other arbitrary infinite Lévy jumps. We particularly focus on three infinite pure jumps Lévy processes that will be added to the stochastic volatility model, which are Variance Gamma processes (VG hereafter) proposed by Madan and Carr [18], Normal Inverse Gaussian processes (NIG hereafter) proposed by Barndorff-Nielsen [19], Classical Tempered Stable processes (CTS hereafter) presented by Rosinski [20]. Noticeably, the CGMY process proposed by Carr et al. [21] is also a special case of infinite activity tempered stable process which is extensively studied by Kim and Rachev [22], Kuchler [23] and Zaevski [24]. Tempered stable processes are the products of the Lévy measure of α stable process multiplying different tempered functions. Although α stable processes can characterize the high peak nature in financial assets, their tail distributions are too thick to capture real tail behaviors in markets. After multiplying tempered functions, they get intermediate between normal distributions and stable distributions appropriately so as to describe the fat tail behaviors of financial data. Then we develop the stochastic volatility Lévy processes (SVLV hereafter) models by subordinating Lévy processes to the stochastic volatility models.

Because model calibrations to market options data formulates a nonlinear optimization problem which often suffers from local minima difficulties, which leads to poor performance in analyzing the behavior of derivative markets, as addressed by Yu [25]. Therefore, it is significant to obtain an accurate parameter set that calibrates the cross-sectional data well for the proposed model so that the model can be effectively applied to pricing options. The same attitude can be found in Yang and Lee [26]. To verify the performance of global intelligent search method on financial models, we apply Differential Evolution (DE hereafter) algorithm to the newly developed complex models, of which it outperforms local search techniques and other intelligent algorithms illustrated in Fastrich et al. [27] and Zhang et al. [28]. Since the consequences of intelligent search methods can explain the observed leptokurtic features for returns, it performs well in empirical researches. Then simulations are carried out exploiting both model prices and Hang Seng index option prices in Chinese Hong kong market case.

The contributions of our work to the previous literatures are that we incorporate both stochastic volatility and infinite Lévy jumps in stock returns simultaneously by subordinating infinite activity Lévy processes to the stochastic volatility models, which are compared to finite jumps stochastic volatility Bates model. The advantage of this newly constructed model framework over the prevailing is the reflection of infinite activity jump behaviors in underlying assets and clustering effect in time changing volatility at the same time. Moreover, the globally intelligent optimization DE algorithm is firstly exploited to calibrate highly dimensional financial models such that the estimation results of SVLV models fitted to option data can be significantly improved, solving the poor performance of nonlinear optimization problems when utilizing local search techniques.

The remainder of the paper is organized as follows. In Section 2 we describe the proposed stochastic volatility Lévy jumps models by subordinating the extensively used infinite activity Lévy processes to the mean reverting CIR stochastic volatility process. In Section 3 we give an outline of Carr–Madan's Fourier transform method and the fast Fourier transform (FFT) technique, the Differential Evolution algorithm is briefly introduced subsequently. Then the empirical researches results are exhibited in Section 4 in which calibrations are performed both to real market data, finally we conclude the paper in Section 5.

2. Stochastic volatility models with infinite activity Lévy processes

2.1. Infinite activity Lévy processes

One can define the Lévy process $X(t)$ in the filtration space (Ω, F, P) an infinite divisible distribution that has independent and stationary increments and nth power of characteristic functions as characteristic functions $\varphi(u)$ themselves. In Sato [29] the infinite divisible Lévy distribution has a triplet of characteristics (μ, σ, υ), in which it respectively represents one of the three independent parts: a linear deterministic part, a Brownian motion part and a pure jump part.