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A comparative study of two-dimensional multifractal detrended fluctuation analysis and two-dimensional multifractal detrended moving average algorithm to estimate the multifractal spectrum

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HIGHLIGHTS

- We study the differences, the advantages and the applicabilities of 2D-MFDFA and 2D-MFDMA.
- Compare 2D-MFDFA and 2D-MFDMA to detect the inheritance and development of DFA and DMA.
- Do the comparative analysis of 2D-MC by the 2D-MFDFA and 2D-MFDMA from six aspects for the first time.
- Provide a guidance on the choice and parameter settings of two methods in the real applications.

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ABSTRACT

Multifractal detrended fluctuation analysis (MFDFA) and multifractal detrended moving average (MFDMA) algorithm have been established as two important methods to estimate the multifractal spectrum of the one-dimensional random fractal signal. They have been generalized to deal with two-dimensional and higher-dimensional fractal signals. This paper gives a brief introduction of the two-dimensional multifractal detrended fluctuation analysis (2D-MFDFA) and two-dimensional multifractal detrended moving average (2D-MFDMA) algorithm, and a detailed description of the application of the two-dimensional fractal signal processing by using the two methods. By applying the 2D-MFDFA and 2D-MFDMA to the series generated from the two-dimensional multiplicative cascading process, we systematically do the comparative analysis to get the advantages, disadvantages and the applicabilities of the two algorithms for the first time from six aspects such as the similarities and differences of the algorithm models, the statistical accuracy, the sensitivities of the sample size, the selection of scaling range, the choice of the q-orders and the calculation amount. The results provide a valuable reference on how to choose the algorithm from 2D-MFDFA and 2D-MFDMA, and how to make the schemes of the parameter settings of the two algorithms when dealing with specific signals in practical applications.

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1. Introduction

Fractals and multifractals are ubiquitous in natural and social sciences [1]. The most usual records of observable quantities are in the form of time series and their fractal and multifractal properties have been extensively investigated. Conventional monofractal analyses numerically define a long-range dependency as a single Hurst exponent, but they assume that the time series are Gaussian distributed [2–6]. Multifractal analyses based on these monofractal analyses have been developed for time series of non-Gaussian distribution [6–13]. Holder exponent is introduced in form of multifractal formalism. This approach describes geometrically and statistically the distribution of singularities on the signal support. Furthermore, the obtained information is much more complete when a multifractal spectrum is estimated. The multifractal spectrum of the studied signal consists in associating to each Holder exponent (local regularity exponent) the dimension of the set of points which exhibit the same value of the Holder exponent [13] . The computation of multifractal analysis is complex. Only a few special sets have analytical expressions. The common sets usually cannot have the analytical expressions . So the multifractal characteristics can only be estimated by numerical computation indirectly.

There are many methods proposed to quantify the statistical properties of monofractal and multifractal signals. These methods can be categorized according to the number of the studied signal and the signal dimension. For one signal of the one-dimensional monofractal signals , there are many methods to extract the Hurst exponent of long-range correlated signal, such as height–height correlation analysis (HHA) [4], autocorrelation function analysis, spectral analysis, rescaled range (R/S) analysis, fluctuation analysis (FA), detrended fluctuation analysis (DFA) [2], detrended moving average (DMA) [3], modified detrended fluctuation analysis (MDFA) [2], multiscale detrended fluctuation analysis (MSDFA) [5], empirical mode decomposition (EMD) and wavelet transform (WT) [2]. For one signal of the one-dimensional multifractal signals, the multifractal analysis methods of statistical physics based on q-order statistic moments mainly include qth order height–height correlation analysis [4], qth order R/S analysis [6], the methods based on structure partition function [7,8] and the multifractal detrending analysis methods. The multifractal detrending analysis methods based on q-order statistics can be divided into two groups: those based on the traditional detrending analysis and those based on the wavelet transform, such as multifractal detrended fluctuation analysis (MFDFA) [9,10], multifractal detrended fluctuation analysis based on empirical mode decomposition ($MFDFA_{EMD}$) [11], multifractal detrended moving average (MFDMA) algorithm [1], the wavelet transform modulus maxima (WTMM) [9] and wavelet leaders (WL) [12,13]. The wavelet leaders approach can be used for signals of arbitrary length while the WTMM is often restricted to much shorter ones. There are two multifractal analysis methods which do not use q-order statistics, they are gradient modulus wavelet projection (GMWP) method and local detrended fluctuation analyses (DFA_{loc}) [11]. To study the long-range cross-correlations of two signals of the one-dimensional time series, the height cross-correlation analysis (HXA) [14], the cross-correlation analysis based on the partition function [8], the detrended cross-correlation analysis (DCCA) [15] and some relative methods [5,16,17] were proposed. A lot of multifractal detrended cross-correlation analysis (MFDCCA) methods [8,14,17–21] have been proposed to discuss the long-range cross-correlations of two signals of the one-dimensional multifractal curves, such as multifractal height cross-correlation analysis (MFHXA) [14], multifractal cross-correlation analysis based on the partition function (MFXPF) [8,18], multifractal detrended cross-correlation analysis based on the detrended fluctuation analysis (MFXDFA) and the multifractal detrended cross-correlation analysis based on the detrending moving average analysis (MFXDMA) [21]. However, the multifractal singularity spectrum does not take the time instant and energy of the one-dimensional time series into account. The time-singularity multifractal spectrum distribution, the fractal energy measurement and the singularity energy spectrum analysis were proposed to make it possible to describe the dynamic evolutionary procedure in the non-stationary and nonlinear systems [22–26].

For the two-dimensional monofractal surfaces, there are several methods: two-dimensional R/S analysis [6], two-dimensional HHA [27], two-dimensional DFA [28] and DMA [29]. For the two-dimensional multifractal surfaces, multifractal analysis methods mainly include the partition function approach based on box-counting method and the multifractal detrending analysis methods. The multifractal detrending analysis algorithms can be categorized into two groups: algorithms based on the traditional detrending analysis and wavelet transform, such as two-dimensional MFDFA [28], two-dimensional MFDMA [1], two-dimensional WTMM [30] and WL [13]. The two-dimensional WTMM and WL methods are more accurate, but the implementations of them are complicated. Multifractal analyses based on the wavelet analyses compute q-order mass exponents firstly and q-order Hurst exponents secondly, but the multifractal analysis methods based on the detrended fluctuation analysis (DFA) and the detrended moving average (DMA) analysis have the opposite steps. The idea of DFA was originally invented to investigate the long-range dependence in coding and noncoding DNA nucleotides sequence [1]. DMA, a method based on the moving average or mobile average technique, was firstly introduced to accomplish accurate and fast estimates of H (Hurst exponent) in order to investigate correlations of the one-dimensional fractional Brownian function (FBM) at different scales [3]. The advantage of DFA and DMA methods is the ease of the implementation. The drawback of DMA is the limited accuracy due to biases and nonstationarities, being these functions are calculated over the entire fractal domain. The advantages of the two-dimensional DFA and DMA [28,29] methods are the ease of the implementation and high accuracy, being calculated over scaled sub-arrays rather than on the whole fractal domain. A further important feature of the two-dimensional DMA algorithm is that it can be implemented “isotropically” or in “directed” mode to accomplish estimates of H in fractals having preferential growth direction, e.g., biological tissues, epitaxial layers, or in crack propagation in fracture. There exist deviations of the estimation of H at small and at large scales. The solution is to increase the size of the fractal surface and do a correction to the detrending analysis by using a weighed