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Interaction between two SU (1, 1) quantum systems and a two-level atom

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HIGHLIGHTS

- The interaction between quantum system and two-level atom is considered.
- The atomic inversion and entanglement are examined.
- The entropy and the variance squeezing, as well as the correlation function are also considered.

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ABSTRACT

We consider a two-level atom interacting with two coupled quantum systems that can be represented in terms of su (1, 1) Lie algebra. The wave function that is obtained using the evolution operator for the atom is initially in a superposition state and the coupled su (1, 1) systems in a pair coherent Barut–Girardello coherent state. We then discuss atomic inversion, where more periods of revivals are observed and compared with a single su (1, 1) quantum system. For entanglement and squeezing phenomena, the atomic angles coherence and phase as well as the detuning are effective parameters. The second-order correlation function displays Bunching and anti-Bunching behavior.

1. Introduction

In nonlinear optical phenomena, one finds that basic nonlinear interactions occur between atomic polarizations and the field via the nonlinearity of the generalized Maxwell–Bloch equations. In this case, the effective Hamiltonian can be derived from a field-dependent nonlinear polarization P through higher order susceptibilities

$$P = \chi_{ij}^{(1)} E_j + \chi_{ijk}^{(2)} E_j E_k + \chi_{ijkl}^{(3)} E_j E_k E_l + \cdots , \tag{1}$$

where $\chi_{ij}^{(1)}$ gives rise to the familiar linear dispersion relation, whereas the term with $\chi_{ijk}^{(2)}$ is responsible for the three-wave mixing processes in parametric devices, as well as second harmonic and subharmonic generation. The term with $\chi_{ijkl}^{(3)}$ is responsible for four-wave mixing processes, which have found considerable applications in connection with phenomena related to optical phase conjugation [1,2]. Moreover it is relevant to stimulated Raman and Brillouin scattering as well as the parametric coupling of Stokes and anti-Stokes radiation. For example, in Brillouin scattering, one finds that an intense

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monochromatic laser source induces parametric coupling between two scattered electromagnetic fields and the acoustical phonons in the scattering medium. In Raman scattering, a similar coupling occurs between the scattered Stokes and anti-Stokes waves and the optical phonons of a Raman-active medium [3–11]. One of the Hamiltonian models which describes four-wave mixing can be written in the form

$$\frac{\hat{H}}{\hbar} = \sum_{i=1}^{2} \omega_i \left(\hat{a}_i^{\dagger} \hat{a}_i + \frac{1}{2} \right) + \lambda \left(\hat{a}_1^2 \hat{a}_2^2 + \hat{a}_1^{\dagger 2} \hat{a}_2^{\dagger 2} \right), \tag{2}$$

where ω_i $(i = 1, 2)$ are the field frequencies, λ is the coupling parameter, whereas $\hat{a}_i^{\dagger}$ and $\hat{a}_j$ are the boson creation and annihilation operators satisfying $\left[\hat{a}_j, \hat{a}_i^{\dagger} \right] = \delta_{ij}$. Note that the interaction term can be regarded as the interaction between two different second-harmonic modes [12,13]. Suppose we inject mixed-four waves within a cavity containing a single two-level atom; this would lead to an interaction between the four waves and the atom (electromagnetic radiation and matter). Consequently, the above Hamiltonian takes the form

$$\frac{\hat{H}}{\hbar} = \sum_{i=1}^{2} \omega_i \left(\hat{a}_i^{\dagger} \hat{a}_i + \frac{1}{2} \right) + \frac{\omega_0}{2} \hat{\sigma}_z + \lambda \left(\hat{a}_1^2 \hat{a}_2^2 \hat{\sigma}_+ + \hat{a}_1^{\dagger 2} \hat{a}_2^{\dagger 2} \hat{\sigma}_- \right), \tag{3}$$

where $\hat{\sigma}_+$ $\left(\hat{\sigma}_-\right)$ and $\hat{\sigma}_z$ are the raising (lowering) and inversion operators, respectively. Note that, these operators satisfy the commutation relations

$$\left[\hat{\sigma}_z, \hat{\sigma}_\pm \right] = \pm 2 \hat{\sigma}_\pm, \qquad \left[\hat{\sigma}_+, \hat{\sigma}_- \right] = \hat{\sigma}_z. \tag{4}$$

Furthermore, in a different context one can find an intensity-dependent form given by [14,15]

$$\frac{\hat{H}}{\hbar} = \omega \left(\hat{n} + \frac{1}{2} \right) + \frac{\omega_0}{2} \hat{\sigma}_z + \lambda \left(\hat{a} \sqrt{\hat{n}} \hat{\sigma}_+ + \sqrt{\hat{n}} \hat{a}^{\dagger} \hat{\sigma}_- \right), \tag{5}$$

where $\hat{n} = \hat{a}^{\dagger} \hat{a}$. Moreover, Eqs. (3) and (5) can be cast in terms of su (1, 1) Lie algebra generators $\hat{K}_+$, $\hat{K}_-$, and $\hat{K}_z$, which satisfy the commutation relations

$$\left[\hat{K}_z, \hat{K}_\pm \right] = \pm \hat{K}_\pm, \qquad \left[\hat{K}_-, \hat{K}_+ \right] = 2 \hat{K}_z, \tag{6}$$

with the corresponding Casimir operator $\hat{K}$ given by

$$\hat{K}^2 = \hat{K}_z^2 - \frac{1}{2} \left(\hat{K}_+ \hat{K}_- + \hat{K}_- \hat{K}_+ \right) \tag{7}$$

that has the eigenvalue $k(k-1)$ with k the Bargmann index. The other operators have the following action on the eigenstate $|m; k\rangle$ of the two operators $\hat{K}^2$ and $\hat{K}_z$:

$$\begin{aligned} \hat{K}_z |m; k\rangle &= (m + k)\, |m; k\rangle, \\ \hat{K}^2 |m; k\rangle &= k(k-1) |m; k\rangle \\ \hat{K}_+ |m; k\rangle &= \sqrt{(m+1)(m+2k)} |m+1; k\rangle, \\ \hat{K}_- |m; k\rangle &= \sqrt{m(m+2k-1)} |m-1; k\rangle, \end{aligned} \tag{8}$$

where $\hat{k}_- |0; k\rangle = 0$. Here, k is the Bargmann index, whereas m is any nonnegative integer.

It is to be mentioned that su (1, 1) Lie algebra can be realized in terms of boson annihilation and creation operators, it is isomorphic to the Lie algebra of the non-compact SU (1, 1) group. Therefore for the Hamiltonian (3), we define $\hat{K}_\pm^{(i)}$ and $\hat{K}_z^{(i)}$, as follows

$$\hat{K}_+^{(i)} = \frac{1}{2} \hat{a}_i^{\dagger 2}, \qquad \hat{K}_-^{(i)} = \frac{1}{2} \hat{a}_i^2, \qquad \hat{K}_z^{(i)} = \frac{1}{2} \left(\hat{a}_i^{\dagger} \hat{a}_i + \frac{1}{2} \right), \quad i = 1, 2 \tag{9}$$

with the Bargmann index k in this case taking either $k = \frac{1}{4}$ (for the even parity states) or $\frac{3}{4}$ (for odd-parity states).

Moreover, in the Holstein–Primakoff representation, these generators can be written for the Hamiltonian (4) in the form [16–18]

$$\hat{K}_+ = \sqrt{\hat{n}} \hat{a}^{\dagger}, \qquad \hat{K}_- = \hat{a} \sqrt{\hat{n}}, \qquad \hat{K}_z = \left(\hat{n} + \frac{1}{2} \right) \tag{10}$$

where the Bargmann index in this case equals 1/2 corresponding to the intensity-dependent model, and hence the discrete basis coincides with the ordinary oscillator number basis.