Available online at www.sciencedirect.com
ScienceDirect
Physica A 387 (2008) 817–824
PHYSICA A
www.elsevier.com/locate/physa

# Multicritical behavior of the two-field Ginzburg–Landau model coupled to a gauge field

L.M. Abreu[a,*], C. de Calan[b], A.P.C. Malbouisson[c]

[a] Centro de Ciências Exatas e Tecnológicas, Universidade Federal do Recôncavo da Bahia, Campus Cruz das Almas, CEP 44380-000, Bahia, Brazil
[b] Centre de Physique Théorique, Ecole Polytechnique, 91128, Palaiseau, France
[c] CBPF/MCT, Rua Dr. Xavier Sigaud, 150, Rio de Janeiro RJ, Brazil

Received 1 April 2007; received in revised form 21 August 2007
Available online 1 October 2007

**Abstract**

In this paper, we study the multicritical behavior of the Ginzburg–Landau model in a $O(n_1) \oplus O(n_2)$-symmetric version containing $(n_1/2 + n_2/2)$-complex order parameters coupled to a gauge field. We develop the RG analysis at a one-loop approximation in the context of the $\epsilon$-expansion approach. The beta functions are obtained, and in the case of equal couplings between the two scalar fields and the gauge field and $n_1 = n_2 = n/2$, the infrared stability of the fixed points is discussed. It is found that the charged infrared-stable fixed point exists for $n > 393.2$. Calculations of the relevant critical exponents are also carried out.


*PACS:* 74.20.-z; 05.10Cc; 11.25.Hf

*Keywords:* Multicritical behavior; Tetracritical point; Renormalization group; $\epsilon$-expansion; Ginzburg–Landau model

## 1. Introduction

Multicritical phenomena appear in systems which present competition among distinct types of ordering. There are several situations in which this kind of phenomenon arises. One typical case is the $^4$He, where the competing order parameters are related to the superfluid and crystalline phases [1]. In anisotropic antiferromagnets in an external uniform magnetic field, the competition is between the parallel and the longitudinal orderings, which depends on the alignment of the magnetic field with the axis of magnetic anisotropy [2–4]. In the context of cuprate-based materials, there are other phases near or simultaneous to the superconducting one, as the antiferromagnetic ordering [5]. See also Refs. [6–9] for other examples of multicritical phenomena.

About three decades ago, the usefulness of renormalization group (RG) analysis in the characterization of multicritical phenomena was shown, in the context of the Ginzburg–Landau model with two order parameters [2–4].

* Corresponding author.
*E-mail address:* abreulm@uol.com.br (L.M. Abreu).


doi:10.1016/j.physa.2007.09.033

It was argued that at first order in the $\epsilon$-expansion, the IR-stable fixed points can have a bicritical or tetracritical behavior, depending on the number of components of the order parameters. Recently, this approach has also been applied to study the phase diagram of high-$T_c$ superconductors [10–14]. In this scenario, five-loop $\epsilon$-expansion computations suggested that the multicritical behavior has a tetracritical nature or is of first-order if the system is outside of the attraction domain of the tetracritical point [15,16].

In this paper, we extend the RG analysis of the $O(n_1) \oplus O(n_2)$-symmetric Ginzburg–Landau model by considering the $U(1)$ gauge-field coupling to the complex scalar fields. This study is developed at the one-loop approximation within the $\epsilon$-expansion approach. The beta functions are obtained. Then, taking the particular case of equal couplings between the two scalar fields and the gauge field, and in addition $n_1 = n_2 = n/2$, the infrared (IR) stability of the fixed points (FP's) is discussed. The RG flow diagrams suggest that the charged IR-stable FP exists for $n > 393.2$. Also, the relevant critical exponents are found.

Let us consider the GL Hamiltonian with a $O(n_1) \oplus O(n_2)$ symmetry, in the presence of an $U(1)$ gauge field in Euclidean $d$-dimensional space,

$$\mathcal{H} = \sum_{\alpha=1}^{2} \left[ |(\partial_\mu - \mathrm{i} e_{0\alpha} A_{0\mu})\phi_{0\alpha}|^2 + r_{0\alpha}|\phi_{0\alpha}|^2 + \frac{u_{0\alpha}}{6}(|\phi_{0\alpha}|^2)^2 \right] + \frac{u_{03}}{3}|\phi_{01}|^2|\phi_{02}|^2 + \frac{1}{4}\left(F_{0\mu}\right)^2 + \frac{1}{2\xi}\left(\partial_\mu A_{0\mu}\right)^2. \tag{1}$$

The zero subscripts denote bare quantities; $\phi_{01}$ and $\phi_{02}$ are complex fields with $n_1$ and $n_2$ real components, respectively, defined by,

$$\phi_{0\alpha} = \begin{pmatrix} \varphi_{0\alpha,1} + \mathrm{i}\varphi_{0\alpha,2} \\ \vdots \\ \varphi_{0\alpha,n_\alpha - 1} + \mathrm{i}\varphi_{0\alpha,n_\alpha} \end{pmatrix}. \tag{2}$$

In our notation, greek indices $\alpha, \beta, \ldots$ denote the types of the scalar fields, i.e. $\phi_1$ or $\phi_2$, and $\mu, \nu, \ldots = 1, 2, \ldots, d$. The term $\left(F_{0\mu}\right)^2 = 2(\nabla \times \mathbf{A}_0)^2$ is the energy of the gauge field. To simplify calculations, we use the Landau gauge $\xi = 0$ (it enforces $\nabla \cdot \mathbf{A}_0 = 0$). Notice the existence of two different gauge couplings. Due to the existence of two order parameters that control the model in (1), the different types of ordering give rise to a multicritical behaviour.

Since the coupling constants have mass dimension $\epsilon = 4 - d$, it is convenient to introduce the dimensionless ones,

$$g_{0i} = u_{0i}\mu^{-\epsilon}; \quad i = 1, 2, 3, \tag{3}$$

$$f_{0\alpha} = e_{0\alpha}^2 \mu^{-\epsilon}; \quad \alpha = 1, 2, \tag{4}$$

where $\mu$ is a mass scale.

## 2. The $\epsilon$-expansion

### 2.1. RG analysis

We consider the RG analysis of the model described by Eq. (1) within the context of the $\epsilon$-expansion. Thus, at one-loop order, we have the following beta functions:

$$\beta_{f_\alpha} = -\epsilon f_\alpha + \sum_{\beta=1}^{2} \frac{n_\beta}{6} f_\alpha f_\beta, \tag{5}$$

$$\beta_{g_\alpha} = -\epsilon g_\alpha + \frac{(n_\alpha + 8)}{6} g_\alpha^2 + \frac{1}{6}\sum_{\beta=1}^{2} \varepsilon^{\alpha\beta}\overline{n}^\beta g_3^2 - 6 f_\alpha g_\alpha + 18 f_\alpha^2, \tag{6}$$

$$\beta_{g_3} = -\epsilon g_3 + g_3 \sum_{\beta=1}^{2} \left[ \frac{(n_\beta + 2)}{6} g_\beta - 3 f_\beta \right] + \frac{2}{3} g_3^2 + 18 f_1 f_2, \tag{7}$$