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Synchronization control of recurrent neural networks with distributed delays

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Abstract

This paper deals with the synchronization problem of the recurrent neural networks with time-varying and distributed time-varying delays. Based on the drive–response concept, LMI approach and the Lyapunov stability theorem, a delay-dependent feedback controller is derived to achieve the exponential synchronization. The derivative of the time-varying delay being less than 1 is released and the activation functions are assumed to be of more general descriptions, which generalize and improve those earlier methods. Finally, two numerical examples are given to demonstrate the effectiveness of the presented synchronization scheme.

Keywords: Exponential synchronization; Recurrent neural networks; Lyapunov functional; Distributed delay; LMI approach

1. Introduction

During the past decade, synchronization of chaotic systems has attracted considerable attention since the pioneering works of Pecora and Carroll [1,2]. They had shown that when some conditions are satisfied, a chaotic system (the slave system/the response system) may become synchronized to another identical chaotic system (the master system/the drive system) if the master system sends some driving signals to the slave one. Now, it is widely known that there exist many benefits of having synchronization or chaos synchronization in some engineering applications, such as secure communication [3], image processing [4] and harmonic oscillation generation. Also, there exists synchronization in language emergence and development, which comes up with a common vocabulary, while agents synchronization in organization management will improve their work efficiency. Recently, chaos synchronization has been extensively investigated due to its potential application in various fields. Specially, artificial neural network models can exhibit chaotic behavior [5–9], and so, synchronization of chaotic neural networks has become an important area of study.

As special complex networks, delayed neural networks have also been found to exhibit complex and unpredictable behaviors including stable equilibria, periodic oscillations, bifurcation and chaotic attractors [10–20]. Some works dealing with chaos synchronization phenomena in delayed neural networks have also been published [21–24]. In Ref. [21], for the delayed chaotic neural networks, a memoryless decentralized control law is proposed which

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guarantees the exponential synchronization when input nonlinearity is presented. In Ref. [22], this paper deals with the synchronization problem of a class of neural networks with delays, and a delay-independent and decentralized control law is derived to achieve the exponential synchronization. By the Halanay inequality lemma, a delay-independent sufficient exponential synchronization condition on chaotic neural networks with delays is derived in Ref. [23]. In Ref. [24], synchronization control of stochastic neural networks with time-varying delays has been considered and a control method is given by using the LMI approach. However, as for synchronization problems in Refs. [21–23], the proposed methods are not presented in terms of LMIs, which makes their checking by the developed algorithms somewhat difficult and inconvenient. The proposed methods in Ref. [24] cannot be applicable when the derivative of time-varying delay equals or is greater than 1. To the best of the authors' knowledge, when derivative of the time-varying delay can take any value, the synchronization problem on the recurrent neural networks with time-varying and distributed delays has not been discussed in terms of LMIs, which remains important and challenging.

Consequently, this paper presents a systematic design procedure for the exponential synchronization of recurrent neural networks with both time-varying and distributed time-varying delays. Based on the drive–response concept and the Lyapunov stability theorem, a memory control law is proposed which guarantees global exponential synchronization, and also it can guarantee the synchronization of the drive–response neural networks with delays. Finally, two illustrative examples are given to show the effectiveness of proposed methods.

The rest of the paper is organized as follows. In Section 2, problem formulations and some preliminaries are introduced. Section 3 deals with the synchronization in an array of delayed neural networks, some sufficient conditions are presented for the synchronization of the delayed drive systems and response ones. Numerical simulations are given to demonstrate the usefulness of the obtained results in Section 4. Finally, concluding remarks are given in Section 5.

Notations. Throughout this paper, for the symmetric matrices $X, Y, X > Y$ (respectively, $X \geq Y$) means that $X - Y > 0 (X - Y \geq 0)$ is a positive-definite (respectively, positive-semidefinite) matrix; $\lambda_{\max}(A)$, $\lambda_{\min}(A)$ denote the maximum eigenvalue and minimum eigenvalue of the matrix A, respectively. A^{T}, $A^{-\mathrm{T}}$ represent the transposes of matrices A and A^{-1}, respectively. The symmetric term in a symmetric matrix is denoted by $*$; i.e.,

$$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix}.$$

2. Problem formulations

Now, consider the following recurrent neural networks with discrete delays:

$$\dot{z}(t) = -Cz(t) + Ag_1(z(t)) + Bg_2(z(t - \tau(t))) + D \int_{t-\varrho(t)}^{t} g_3(z(s))\mathrm{d}s + I(t), \tag{1}$$

where $z(t) = [z_1(t), \ldots, z_n(t)]^{\mathrm{T}} \in \Re^n$ is the neuron state vector; $g_i(z(\cdot)) = [g_{i1}(z_1(\cdot)), \ldots, g_{in}(z_n(\cdot))]^{\mathrm{T}} \in \Re^n, i = 1, 2, 3$ represent neuron activation functions; $I(t) = [I_1(t), \ldots, I_n(t)]^{\mathrm{T}} \in \Re^n$ is a time-varying input vector; $C = diag(c_1, \ldots, c_n)$ is a diagonal matrix with $c_i > 0$; and A, B, D are the connection weight matrix, the delayed weight matrix and the distributively delayed connection weight matrix, respectively; here, $\tau(t), \varrho(t)$ denote the time-varying delay and the distributed time-varying one satisfying

$$0 \leq \tau(t) \leq \tau_m, \qquad \dot{\tau}(t) \leq \mu, \qquad 0 \leq \varrho(t) \leq \varrho_m, \tag{2}$$

and τ_m, μ, ϱ_m are constants. The initial conditions of (1) are given by $z_i(t) = \psi_i(t) \in C([-\tau_{\max}, 0], \Re)$, where $C([-\tau_{\max}, 0], \Re)$ denotes the set of all continuous functions from $[-\tau_{\max}, 0]$ to $\Re$. Here $\tau_{\max} = max\{\tau_m, \varrho_m\}$.

Remark 1. For the synchronization tasks addressed in Refs. [22,24], the derivatives of delays considered are less than 1. However, the delays in the paper are time-varying ones and one of their derivatives can take any value, which is more meaningful than the ones in Refs. [22,24].

The following assumption is made on the neuron activation functions.

Assumption 1. For $i \in \{1, 2, \ldots, n\}$, the neuron activation functions in (1) are bounded and satisfy