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Optimal traffic networks topology: A complex networks perspective

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Abstract

We investigate and analyse an optimal traffic network structure for resisting traffic congestion with different volumes of traffic. For this aim, we introduce a cost function and user-equilibrium assignment (UE) which ensures the flow balance on traffic systems. Our finding is that an optimal network is strongly dependent on the total system flow. And the random network is most desirable when the system flow is small. But for the larger volume of traffic, the network with power-law degree distribution is the optimal one. Further study indicates, for scale-free networks, that the degree distribution exponent has large effects on the congestion of traffic network. Therefore, the volume of traffic and characteristic of network determine the optimal network structure so as to minimize the side-effect produced by traffic congestion.

Keywords: Optimization; Traffic congestion; UE assignment; Scale-free networks; Random networks

1. Introduction

Many real-world systems take the form of networks — nodes or "vertices" joined together by edges or "connections". Commonly cited examples include communication networks, information networks, traffic and transportation networks, distribution networks, and other naturally occurring networks. The beginning of the study of topology of complex networks started from the random graph theory of Erdös and Rényi (ER) [1]. Triggered by two seminal papers [2,3], complex networks have attracted a great deal of attention in recent years. Especially in Ref. [3], Barabási and Albert demonstrated that, with the algorithm of growth and preferential attachment, the probability $p(k)$ that a vertex in the network is connected to k other vertices decays as a power-law (or the fraction of nodes in the network that have degree k), following $p(k) \sim k^{-\lambda}$, where $\lambda = 3$ is the degree distribution exponent. This power-law distribution of a node's degree, defined as the number of its next neighbors, is the characteristic of scale-free (SF) networks, meaning that structure and dynamics of the network are strongly affected by nodes with a great number of connections. The highly connected "hub" nodes of a scale-free network and the short paths in a strongly clustered small-world greatly facilitate the propagation of an infection over the whole network, which has to be taken into account for designing effective vaccination strategies [4,5].

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Recently, the flow properties of the transported entities (such as traffic flows, information, energy, chemicals, etc.) become of primary interest in complex networks. In particular, traffic congestion and its dynamical relation to network structures have become a hot topic [6,7]. There have been many previous studies to find the optimal topology [8,9], and understand or control the traffic congestion on complex networks [10–18]. In Ref. [19], a general framework had been established considering gradients generated flows, and the problem of jamming in the networks was addressed [19,20]. In this framework, each node in the network was guided by the local gradients at nodes. Their finding is that random networks are more susceptible to jamming than scale-free ones. An interesting issue, then, concerns the jamming properties of random and scale-free networks with same parameters $\langle k \rangle$ and N. In Ref. [21], Toroczkai and Bassler studied that jamming was limited in scale-free networks. Other researchers analysed the conditions that jamming could occur in different network topologies and the dynamic behaviors caused by traffic jamming [22]. During the past few years, there has much interest in the optimal network topology, such as the two-peak and three-peak optimal complex networks. Few literature concerted on the congestion and cascade in complex networks had been studied in Refs. [23–27]. In fact, flows on edges are limited and are affected by the topology of the network. When the flows on a link exceeds its capacity, congestion will occur. Moreover, little effort has been made on which traffic network topology is the optimal one and which one will lead to the smallest congestion under the condition of different traffic flows. Therefore, this paper want to solve these two fundamental questions: Which traffic topology is the optimal one and which topology characteristics lead to the smallest degree of congestion when the congestion effect is considered?

2. Generation of artificial traffic network and some definitions

We represent networks as graphs $G = (V, E)$ where V is the set of vertices, and E is the set of edges. G is described by the $N \times N$ adjacency matrix $\{e_{ij}\}$. Define N as the size of the network. If there is a link between nodes i and j, the entry e_{ij} is the value 1; otherwise $e_{ij} = 0$. We start by constructing networks according to ER algorithms, which have been mentioned in Ref. [1]. The Molloy–Reed algorithm [28] is used to generate scale-free networks with different exponents. In order to guarantee the existence of a single connected cluster, we check the largest connected component at each iteration. If the largest connected component is equivalent to the size of the network, we will perform the next iterance. Otherwise, pairs of end-links are chosen at random and connected to form a link again. To introduce dynamical congestion, we assign a capacity U_{ij} on the edge (i, j), which shows the maximum possible crossing flows on the edge (i, j). In order for an edge to function properly, its flow must meet the equation $Q_{ij} \le U_{ij}$ at all times; otherwise the edge is considered being congested, where Q_{ij} denotes the flows on the corresponding edge [24]. In all cases, we set the size of network $N = 100$ (While being quite small due to computational constraints and the time complexity of UE assignment, this does provide a sufficiently sized network to gain statistically significant attributes.) and the average degree is $\langle k \rangle = 2.7$.

Definition 1. The *degree* (or connectivity) k_i of a node i is the number of edges incident with the node.

Definition 2. A measure of the typical separation between two nodes in the graph is given by the *average shortest path length*, also known as characteristic path length, defined as the mean of geodesic lengths over all couples of nodes [2]:

$$L = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij},$$

where the d_{ij} is the length of the geodesic from node i to node j.

Definition 3. The node (edge) *betweenness* is defined as the number of shortest paths between pairs of nodes that run through that node (edge) [29].

Definition 4. The *Efficiency* F is always used to measure how efficiently information is exchanged in the network. Latora and Marchiori defined the global efficiency as follows [29]:

$$F = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}},$$

where d_{ij} is the shortest path length between two generic vertices i and j.