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Potts model on directed small-world Voronoi–Delaunay lattices

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HIGHLIGHTS

- $q = 3$ state in Potts model has a different universality class for the second order phase transition when $p = 0.01$.
- For $p > 0.01$ in the $q = 3$ state, the phase transition is first order.
- For the $q = 4$ state the phase transition is first order for any value of the rewiring probability p.

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ABSTRACT

The critical properties of the Potts model with $q = 3$ and 4 states in two-dimensions on *directed* small-world Voronoi–Delaunay random lattices with quenched connectivity disorder are investigated. This disordered system is simulated by applying the Monte Carlo update heat bath algorithm. The Potts model on these *directed* small-world random lattices presents in fact a second-order phase transition with new critical exponents for $q = 3$ and value of the rewiring probability $p = 0.01$, but for $q = 4$ the system exhibits only a first-order phase transition independent of p $(0 < p < 1)$.

1. Introduction

Experimental studies have shown that the presence of impurities or inhomogeneities can affect the critical behavior of magnetic materials [1]. From a theoretical point of view, the effect of the impurities in magnetic systems was proposed by Harris [2] studying physical system with some kind of disorder. For quenched disorder the Harris criterion [2] is an important tool in the theoretical understanding of the critical behavior of physical systems with impurities or inhomogeneities. In this particular case, the randomness can be classified by the sign of the specific heat exponent of the pure system, α_{pure}. For $\alpha_{pure} > 0$ the quenched random disorder is a relevant perturbation, leading to a critical behavior different from the pure case. For $\alpha_{pure} < 0$ disorder is irrelevant, and for the marginal case $\alpha_{pure} = 0$ no prediction can be made. For first-order phase transitions it is well known that the influence of quenched random disorder can lead to a softening of the transition [3]. This predicted softening effect at first-order phase transitions has been confirmed in three-dimensions ($D = 3$) using Monte Carlo [4–6], high temperature series expansion [7] techniques, and in two-dimensions ($D = 2$) where several models with $\alpha_{pure} > 0$ [8–11] and the marginal ($\alpha_{pure} = 0$) [12–16] have been studied.

In this paper we study the effect of *directed* bound case with rewiring probability p [17]. Here, we consider $D = 2$ small-world Voronoi Delaunay random lattices (SWVD) type, and perform an extensive computer simulation study of the

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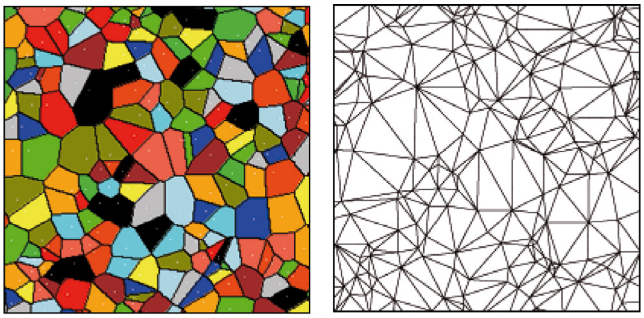

Fig. 1. (Color online) (a) A patch of a Voronoi diagram. (b) The corresponding dual lattice to the diagram shown in (a), courtesy of Oliveira et al. [18].

Potts model. We applied finite-size scaling (FSS) techniques to extract the exponents and the "renormalized charges" U_4^*. Monte Carlo simulations of the disorder system were performed using the spin-flip heat bath algorithm to update the spins. Previous studies of connectivity disorder in $D = 2$ lattices have been done by Monte Carlo simulations of q-state Potts models on quenched random lattices of Voronoi Delaunay (VD) [18] type for $q = 2$ [19–21], $q = 3$ [22] and $q = 8$ [23,24]. In particular, it has been shown that for $q = 2$ [19–21] and $q = 3$ [22,25] the critical exponents are the same as those for the model on a regular $D = 2$ lattice. This is indeed a surprising result since the relevant criterion of the Delaunay triangulations reduces to the well known Harris criterion. A disorder of this type should be relevant for any model with positive specific heat exponent [26]. This means that for $q = 3$, where $\alpha_{pure} > 0$, one would expect a different universality class. For the spin-1 Ising model, where $\alpha_{pure} = 0$, Fernandes et al. [27,28] showed that the exponents do no change in the VD lattice, but for *directed* SWVD random lattice the situation is quite different. There is a second-order phase transition for $p < p_c$ and a first-order phase transition for $p > p_c$, where $p_c \approx 0.35$ is the rewiring probability where the system changes the phase transition order. In addition, the calculated critical exponents for $p < p_c$ do not belong to the same universality class as the regular two-dimensional ferromagnetic model. Therefore both VD and *directed* SWVD random lattice cases agree with Harris criterion for $\alpha_{pure} = 0$.

In the present Potts model with $q = 3$ and 4 states on *directed* SWVD lattice we show that the critical behavior for $q = 3$ is quite similar to that observed by Fernandes et al. in the spin-1 case [27,28] and for $q = 4$ only a first-order phase transition was found for $(0 < p < 1)$. In the next section we present the model and the simulation background. The results and conclusions are discussed in the last section.

2. Model and simulation

In the VD random lattice the construction of the lattice obeys the following procedure: for each point in a given set of points in a plane, we determine the polygonal cell that contains the region of space nearest to that point than any other. Two cells are considered neighbors when they possess an extremity in common (Voronoi diagram). From this Voronoi diagram, we can obtain the dual lattice, or triangulation of Delaunay (Delaunay tessellation), by the following procedure, see Fig. 1. (i) When we have two neighbor cells, a link is placed between the two points located in the cells. (ii) From the links, we obtain the triangulation of space that is called the Delaunay lattice. (iii) The Delaunay lattice is dual to the Voronoi diagram in the sense that its points correspond to cells, links to edges and triangles to the vertices of the Voronoi diagram.

After procedure above the build of the *directed* SWVD network is as follows: with probability p, we reconnect nearest-neighbor outgoing links to a different site chosen at random. After repeating this process for every link, we are left with a network with a density p of SWVD links. Therefore, with this procedure every site will have k (where $3 \leq k < 20$ and different for each network site) outgoing and incoming links.

In the case of *directed* SWVD every site will have k outgoing links and varying (random) number of incoming links [18,17,29]. Therefore, here there is not always the reciprocity between the outgoing and incoming links.

We consider the ferromagnetic Potts model with $q = 3$ and 4 states, on *directed* SWVD random lattice where spins variables S_i taking the values $S = 1, 2, 3$ and $S = 1, 2, 3, 4$ for $q = 3$ and 4, respectively, are located on every site i of a *directed* SWVD random lattice with $N = L \times L$ sites, where L is the side of a square lattice. The time evolution of the system is given by a single spin-flip like dynamics with a probability P_i given by

$$P_i = 1/[1 + \exp(2E_i/k_B T)], \tag{1}$$

where T is the temperature, k_B is the Boltzmann constant, and E_i is the energy of the configuration obtained from the Hamiltonian

$$H = -J \sum_{i=1}^{N} E_i, \tag{2}$$