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Persistence and extinction for a class of stochastic SIS epidemic models with nonlinear incidence rate

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HIGHLIGHTS

- Stochastic SIS model with nonlinear incidence rate is established by perturbing transmission coefficient.
- The threshold value $\widetilde{R}_0$ is obtained to determine extinction and weak permanence of the disease in probability.
- The sufficient condition on permanence in the mean of the disease with probability one is showed.
- Numerical simulations are presented to illustrate some open problems.

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ABSTRACT

In this paper, a class of stochastic SIS epidemic models with nonlinear incidence rate is investigated. It is shown that the extinction and persistence of the disease in probability are determined by a threshold value $\widetilde{R}_0$. That is, if $\widetilde{R}_0 < 1$ and an additional condition holds then disease dies out, and if $\widetilde{R}_0 > 1$ then disease is weak permanent with probability one. To obtain the permanence in the mean of the disease, a new quantity $\widehat{R}_0$ is introduced, and it is proved that if $\widehat{R}_0 > 1$ the disease is permanent in the mean with probability one. Furthermore, the numerical simulations are presented to illustrate some open problems given in Remarks 1–3 and 5 of this paper.

1. Introduction

As is well known, our real life is full of randomness and stochasticity. So there are more real benefits to be gained in using stochastic models. Particularly, stochastic models can provide us some additional degrees of realism in comparison to their deterministic counterparts. The relative descriptions can be found in Refs. [1–4].

There are different possible approaches which result in different effects on a population system or epidemic system to include randomness and stochasticity in the models. Typically, the following three approaches are seen most often. The first one is to directly perturb parameters of deterministic model by Gaussian white noise (see Refs. [4–19]). The second one is to assume that stochastic perturbations are around positive equilibrium of deterministic models (see Refs. [19–26]). The last one is the assumption that systems or models will switch from one regime to the other according to the probability law of the Markov chain (see Refs. [27–29]).

In recent years, various stochastic versions of epidemic models are established by using the method of parameters perturbation, and the dynamical properties of these models are also widely investigated. The main research subjects include

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the existence and uniqueness of positive solution with any positive initial value in probability, the persistence and extinction of the disease in probability, the asymptotical behaviors in probability around the disease-free equilibrium and the endemic equilibrium of the corresponding deterministic models, the existence and uniqueness of stationary distribution as well as ergodicity, etc. Many important results have been established in many articles, for example, see Refs. [4–28] and the references cited therein. Especially, in terms of the persistence and extinction for the disease, articles [9,10,12–15,17–19] provided for us some very valuable conclusions. In Ref. [9], a stochastic SIS epidemic model with constant population size is constructed. The authors not only obtained the existence of the unique global positive solution with any positive initial value, but also established threshold value conditions for extinction and persistence of the disease. Furthermore, in the case of the persistence, the authors also showed the existence of a stationary distribution and finally computed the mean value and variance of the stationary distribution. In Ref. [12], a class of stochastic SIS epidemic models with bilinear incidence and vaccination is studied. The authors established the sufficient conditions for extinction and persistence in the mean of the disease. In Refs. [13–15,17], authors studied a class of stochastic SIRS epidemic models with the bilinear incidence or saturation incidence given by function $\frac{\beta SI}{1+\omega_1 S+\omega_2 I}$ (see model (3) in Ref. [13], model (2) in Ref. [14], model (7) in Ref. [15] and model (1.2) in Ref. [17]). The sufficient conditions for the existence and uniqueness of global positive solution, the extinction and persistence in the mean of the disease are investigated. In Ref. [18], the authors proposed a class of impulsive periodic stochastic SIR epidemic models with bilinear incidence, sufficient conditions for the existence and uniqueness of global positive solution and the disease-free periodic solution, the extinction and weak persistence in the mean of the disease are obtained. In Ref. [19], A class of stochastic SIQS epidemic models with nonlinear incidence is studied. The sufficient conditions for the existence of global positive solutions, the extinction of disease and the existence of a unique stationary distribution are established.

Motivated by the above works, in this paper, we consider the following deterministic SIS epidemic model with nonlinear incidence rate

$$\begin{cases} \dfrac{dS(t)}{dt} = & \Lambda - \beta S(t)g(I(t)) + \gamma I(t) - \mu S(t), \\ \dfrac{dI(t)}{dt} = & \beta S(t)g(I(t)) - (\mu+\gamma+\alpha)I(t). \end{cases} \tag{1}$$

In model (1), $S(t)$ and $I(t)$ denote the numbers of susceptible and infectious individuals at time t, respectively. Λ is the recruitment rate of S, μ is the natural death rate of S and I, α is the disease-related death rate of I, γ is the recovery rate of I. The transmission of the infection is governed by a nonlinear incidence rate $\beta Sg(I)$, where β denotes the transmission coefficient between compartments S and I, and $g(I)$ is a continuously differentiable function of I. All parameter values are assumed to be nonnegative and $\Lambda, \mu > 0$.

Now, we assume that the fluctuations in the environment will manifest themselves mainly as fluctuations in the transmission coefficient β of disease, that is, $\beta \to \beta + \sigma\dot{B}(t)$, where $B(t)$ is an one-dimensional standard Brownian motion defined on some probability space and parameter $\sigma > 0$ represents the intensity of $B(t)$. Thus, model (1) will be changed into the following stochastic SIS epidemic model with nonlinear incidence rate

$$\begin{cases} dS(t) = & [\Lambda - \beta S(t)g(I(t)) + \gamma I(t) - \mu S(t)]dt - \sigma S(t)g(I(t))dB(t), \\ dI(t) = & [\beta S(t)g(I(t)) - (\mu+\gamma+\alpha)I(t)]dt + \sigma S(t)g(I(t))dB(t). \end{cases} \tag{2}$$

In this paper, we will discuss the long-time dynamical behaviors of model (2). Particularly, as the main purpose, we will investigate the extinction, weak permanence and permanence in the mean of disease with probability one, and establish the corresponding sufficient conditions. Furthermore, we will validate the main conclusions obtained in this paper by the numerical simulations.

The reminder of this paper is organized as follows. The next section introduces the preliminaries and some useful lemmas. Section 3 investigates sufficient conditions for extinction of the disease in model (2) with probability one. In Section 4, we establish sufficient conditions which ensure that the disease in model (2) is weakly permanent and permanent in the mean with probability one. Then, in Section 5, numerical simulations are carried out to illustrate the main theoretical results. Finally, a brief discussion is given in the end to conclude this work.

2. Preliminaries

Denote $R^2_+ = \{(x_1, x_2) : x_1 > 0, x_2 > 0\}$, $R_{+0} = [0, \infty)$ and $R_+ = (0, \infty)$. Throughout this paper, we assume that model (2) is defined on a complete probability space $(\Omega, \{F_t\}_{t\geq 0}, P)$ with a filtration $\{F_t\}_{t\geq 0}$ satisfying the usual conditions, that is, $\{F_t\}_{t\geq 0}$ is right continuous and F_0 contains all P-null sets.

For function $g(I)$ we further introduce the following assumption

(H) $g(I)$ is continuously differentiable on R_{+0}, $\frac{g(I)}{I}$ is monotone decreasing on R_+, $g(0) = 0$ and $g'(0) > 0$.

Under assumption (H), it is clear that $g(I)$ is Lipschitz continuous on R_{+0} and $0 < g(I) \leq g'(0)I$ for all $I > 0$.