Physica A 451 (2016) 560–564

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

"Buoyancy" in granular medium: How deep can an object sink in sand?

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HIGHLIGHTS

- Buoyancy effect in granular systems.
- Measured the sinking depth (SD) of the ball into granules of different sizes.
- SD is very sensitive to the size of granular system.
- Ratio does not depend on the density of the ball and the volume fraction of granules.
- Empirical formula of SD is obtained.

ARTICLE INFO

Article history:
Received 19 October 2015
Received in revised form 13 January 2016
Available online 10 February 2016

Keywords:
Granular buoyancy
Sinking depth
Sand

ABSTRACT

The behavior of granular matter is different from either fluids or solids. One may not be able to answer even a naive question such as how deep an object can sink in sand. Answers to the depth of footprints on sand beach and its dependence on grain size have never been seriously studied before, and may deserve a closer look and better understanding. Laying a ball of fixed size onto granules, we have measured the sinking depth (*SD*) of the ball into granules of different sizes and studied the dependence of *SD* on the sizes of the ball and granules. We find that the *SD* is very sensitive to the size of granules and the variation of *SD* on granule size is not monotonic. The maximum *SD* occurs at $r \approx \frac{1}{20}R$, where r and R are the radii of granules and the ball, respectively. This ratio does not depend on the density of the ball and the volume fraction of granules. An empirical formula of *SD* on densities and sizes of the ball and granules are obtained based on the experimental results.

1. Introduction

Granular materials have recently become a popular subject of studies [1–3] because of their scientific and technological importance. Our understanding of properties of granular materials is far from complete, such as jamming transition in static granular assemblies [4], a static assembly of non-cohesive, spherical particles in contact [1,5]. Particles within this system are under stress, supporting the weight of the material above them in addition to any applied load. It is known that stresses in a granular assembly are distributed in a highly inhomogeneous manner, along networks containing the largest inter-particle forces known as force chains [6,7]. The assembly will behave like a solid when the force chains are strong enough to hold

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http://dx.doi.org/10.1016/j.physa.2016.02.004
0378-4371/

the assembly together and will flow like a fluid when these force chains are broken under stress [8]. Obviously, properties of these contact force chains crucially determine the bulk properties of the assembly, especially its load bearing capability [9].

The formation of the force chains is the result of the inherently nonlinear nature of inter-particle friction forces and the particle's nearly hard-sphere interaction [10,11]. The forces between granules are characterized by their contacts as they vanish identically when the granules are not in contact [12,13,3,14]. An important characteristic of the force chains is that their distribution in the granular assembly will be modified in response to an external stress [11,15]. When under stress, the particles start deforming. New contacts are created, and additional force chains are created. For that reason, a quantitative understanding of the force chains will be needed to understand the response of the granular assembly to an external stress. However, detailed studies of changes in these contacts and their corresponding forces are proved to be difficult because forces measurement inside a granular assembly is still not possible even if the stress pattern has been observed in a two-dimensional granular material under compression [16,17].

One of the most important findings of these studies is that the force chains support most of the external load, effectively shielding large regions of the material [18]. Presumably, the characteristics of the force chains can be inferred from the external response of the granular assembly to a well-defined external stress without the need of measurements in the interior of the assembly if a reasonable model can be used [11,19–23]. For example, when an intruder is lowered into a granular bed, properties of the contact force between the intruder and the bed will be related to the sinking depth of the intruder, which is the response of the bed [24,25]. In this letter, we report results of such an experiment. Properties of contact forces F in granular beds with beads of size r and density ρ_r are probed by experiments in which the sinking distance d of an intruder of size R and density ρ_R is measured as a function of r when the intruder is lowered into the beds under its own weight. It is found that d is not monotonic and its maximum sinking can be understood as the balance between the changes in F and the number of contacts as r is varied. The ratios r/R and ρ_r/ρ_R are found to be important parameters, and the contact forces are highly nonlinear in nature.

2. Experimental setup

Our experimental setup is very simple. As is shown in the inset of Fig. 1(a), a cylindrical container of an inner diameter 72 mm and a height 50 mm is filled with beads. Directly above the granular bed is an intruder ball, which is suspended from a vertical translation stage so that the position of the intruder can be controlled. The granular beds studied are all disordered three-dimensional, compact random packing of smooth glass beads (spheres) with 21 different radii in a range of $r = 0.02$–1.8 mm with a size dispersion of about 20%. The density of the glass beads is $\rho_r = 2.5$ g/cm^3. Eight radii of intruder balls (stainless steel balls used in ball bearing) are used: 1.50, 2.55, 3.15, 4.00, 5.00, 5.75, 7.50 and 10.30 mm. Density of the intruder balls is $\rho_R = 7.8$ g/cm^3. In order to maintain a constant bulk packing density of the beds for various bead sizes, we always keep the volume of the granules and the total weight constant. The total mass in the experiment is kept a constant of 370 g. The volume of the granular beds is then kept constant by various operations such as shaking, removal and addition of beads. In the experiments, the volume fraction is kept as 0.61 ± 0.01. During the compaction of the granular bed in the procedure above, the surface of the bed is always leveled off. The intruder ball is suspended from a fine cotton cord and lowered onto the surface of the granular packing as slowly as possible. The response of the beds to an intruder lowered into the beds under gravity is measured by the maximum distance that the intruder can travel after the intruder is in contact with the bed. This distance is defined as the *SD* of the intruder. The initial (contact with the bed) and final positions of the intruder are measured by a traveling microscope. The measured d will be a function of the dimension of the container for the granular bed if the container is not large enough to eliminate boundary effects. The container used in our experiment is tested to be large enough by comparing measured d with a larger container. All measurements reported in this report are averaged over ten times to minimize statistical error. It should be pointed out that the intruder ball should be large and heavy enough when compared to the weight of the beads because there is a threshold value of *SD* for one kind of bed (Fig. 3). Also the intruder should not be small to compare with the size of the beads, otherwise big measurement-error will result.

3. Results and discussion

Fig. 1(a) is the measured *SD*, shown as d, for an intruder in granular beds as a function of bead sizes r with different intruder radius R. (Due to too many sets of data in the figure, the error bars are not shown. Error bars are shown in Fig. 2.) When r is small, d increases with r. After reaching a maximum $d_{\max}$, d begins to decrease at larger r. A remarkable feature of Fig. 1(a) is that the measured d is not a monotonic function of the bead sizes. There is a specific value of bead size $r*$ at which d is maximum or the bed seems to be weakest.

When we re-plot the ratio of d/R versus r/R in Fig. 1(b), the curves of different R almost overlap. This indicates d (at a fixed r) and $d_{\max}$ increase linearly with R, and that $d_{\max}$ is found at a characteristic $\lambda = r/R \approx 0.05$. In other words, the sinking depth of an intruder reaches a maximum when its size is about 20 times of the size of the beads. In our experimental condition, $d_{\max}$ is about $0.6R$. In order to confirm $d_{\max}$ and d change consistently with r, the *SD* is measured as a function of r at two different ball densities 10.6 and 5.4 g/cm^3 and under two different packing densities $\phi = 0.57$ and $\phi = 0.61$ as shown in Fig. 2(a) and (b). The results show that for intruder balls of the same size R, the ratio of $d_{\max}/d$ is basically not a function of r when changing the ball density or the bead packing density. In other words, the geometric dependent factor of